A1. The payoff of a strike-K2 call is always larger than that of a K2 (≥ K1) call. Hence, \( C(K2) > C(K1) \) would create an arbitrage opportunity - hence \( C(K1) ≥ C(K2) \).

A2. Payoff of long K1 call, short K2 call:

\[
\begin{align*}
&K_2-K_1 \\
&K_1 & K_2
\end{align*}
\]

Payoff dominated by \( K_2-K_1 \), hence price of portfolio must be ≤ present value of \( K_2-K_1 \) - and hence ≤ \( K_2-K_1 \) (positive interest).

B1. Two-price = \( e^{-0.08}(96.4 - 2 \times e^{-0.04/12}) = 96.823 \)

B2. FWD-price = \( \frac{(96.4 - 2 \times 0.04/12)}{P_{0/12}} \)

Strategy:

1. Buy \( 0.25 \) 8.5 ZCB, sell \( 0.50 \) 2.5 ZCB.

\[ S(6) \times P_{12} \]

\[ P_{12} \]

\[ T = 0.5 \text{ year} \]

\[ \text{Short FWD} \text{ not cashflow} = 0 \]

\[ \text{Short FWD clears ZCB payment} \]

\[ \text{Payoff} = S(T) - \text{FWD} - S(6) \times P_{12} \]

\[ \text{known at time} \rightarrow \text{must} = 0 \text{ for arbitrage} \]

B3. Shorting house price produces will give you positive cashflows when house price falls - a hedge.

B4. No, the UK housing duplex does not correlate perfectly with your house price. Thus important since could use a min.-var. house via a Hull-Jacobi-Brenner hedge.

C1. \( 0.9704 \times 1.04 = 1.0097 \sqrt{0.9704 \times 5 + 0.9246 \times 105} = 1.0194 \sqrt{0.9704 \times 5 + 0.9246 \times 15 + 68 \times 0.8641} = 1.0021 \sqrt{0.9704 + 0.9246 + 0.8641} + 1.04 \times 0.7928 = 0.9349 \sqrt{
\[ f_t = \frac{P_t}{P_{ly}} \quad f_t = (0.03, 0.05, 0.07, 0.09) \]

**C3**

**MUST SOLVE**

\[ 100.21 = \sqrt[3]{37YTM \times 5} + 6.0 \times (1 + YTM)^{-3} \]

If price = 100, YTM = 5%; thus YTM must be a little

under 5% — treat as greek \( \approx \) YTM = 4.93%

**C4**

\[ c = \frac{100}{\sqrt{210.05}} = 53.78 \]

Price annuity = 53.78 \((0.9709 + 0.9246) = 101.94 \)

**C5**

As Price(Annu) < No-Arb price we must buy annuity

and can replicate w/ 1- and 2-year bullets

\[
\begin{align*}
&x_1 \times 6.4 + x_2 \times 5 = -53.78 \quad \begin{cases} x_1 = -0.4925 \text{ short} \\
x_2 = -0.5122 \text{ positive} \end{cases} \\
&0 + x_2 \times 105 = -53.78 \quad \begin{cases} x_1 = -0.4925 \text{ short} \\
x_2 = -0.5122 \text{ positive} \end{cases}
\end{align*}
\]

Future cashflows net out today we receive 0.94 / APR 6

**D1**

\[ PV_{\text{LiA}} = 5 \times a_{20|1.04} = 67.95 \quad (\text{atm}) \]

\[ \text{Dur}_{\text{LiA}} = \frac{\text{(IA) \times D}}{PV_{\text{LiA}}}, \quad \text{IA} = \frac{(1+c) a_{t|c} - T \cdot V_n}{c} \]

\[ D = 5 \quad c = 0.04, \quad v = (1 + i)^5, \quad m = 20 \quad \Rightarrow \text{Dur} = 9.21 \quad (\text{YRABS}) \]

**D2**

**MUST SOLVE**

\[ P_3 \times x_1 + P_5 \times x_2 = PV_{\text{LiA}} \quad (P_j = (1 + 6.4)^{-j}) \]

\[ P_3 \times 5 \times x_1 + P_5 \times 6 \times x_2 = PL_{\text{LiA}} \times D_{\text{LiA}} \]

\[ = D \begin{cases} x_1 = 47.87 \times 10^6 \\
x_2 = 51.51 \times 10^6 \end{cases} \]

**X 2 CBS to buy**
Assets are less dispersed than liabilities, so
the 3rd command for production immunisation is
uti 
(\text{Spec. command (assets) } = 115\%)
(\text{conv. command (liabilities) } = 126) \]

\[ E_1 \quad E(S_{15}) = (1+\mu)^{15} \times 1000 = \$62.36m \quad (\mu = 0.05) \quad (\sigma = 0.1) \]

\[ \text{SD}(S_{15}) = 23.3m \times \sqrt{(1+\mu)^{25} - (1+\mu)^{25}} = \$23.8m \]

\[ E_2 \quad \ln(1+i)^{\text{N} (\mu, \sigma^2)} \]

\[ 1+i = E(1+i) = E(e^{\ln(H_1)}) = e^{\mu + \sigma^2/2} \]

\[ s^2 = \text{VAR}(e^{\ln(H_1)}) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) \]

\[ \sqrt{\frac{s^2}{\sigma^2}} = \frac{S}{\sigma} \quad \text{so} \quad \sigma = \sqrt{\ln(1 + \frac{S^2}{\sigma^2})} = 0.0950 \]

\[ M = \ln(1.05) - \sigma^2/2 = 0.0443 \]

\[ E_3 \quad \text{Prob} (S_{15} \leq 50m) = \text{Prob} \left( \frac{15 \ln(1+H_{15})}{\sigma \sqrt{15}} \leq \ln(1.5) \right) \]

\[ = \text{Prob} \left( \frac{X - 15\mu}{\sigma \sqrt{15}} \leq \frac{\ln(1.5) - 15\mu}{\sigma \sqrt{15}} \right) = \Phi(-0.417) = 0.339 \]