MATH2510

This paper consists of 3 printed
pages, each of which is identified by the reference MATH2510

Only approved basic scientific
calculators may be used in this examination.

Template Exam for MATH2510 for December 8 Workshops
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Financial Mathematics 2

Time allowed (on the real exam): 2 hours and 30 minutes

Answer all questions. Briefly and quickly.
The 17 numbered subquestions (from A1 to E3) are given equal weight in the grading.
(So each subquestion is worth 5% of the total, final grade for the course.)
You may round final answers to the nearest penny or basis point.

QUESTION A

In this question $C(K)$ denotes the price of a strike-$K$ call-option (at some point in
time for some expiry-date and with some underlying, neither of which concern us much.).

A1. Show that $K_2 > K_1 \Rightarrow C(K_2) \leq C(K_1)$ — or else there is arbitrage.

A2. Assume that interest rates are positive. Show that $K_2 > K_1 \Rightarrow K_2 - K_1 \geq C(K_1) - C(K_2)$ — or else there is arbitrage. (Hint: Look at the pay-off profile/function of a portfolio that is long a strike-$K_1$ call-option and short a strike-$K_2$ call-option.)

CONTINUED...
QUESTION B

B1. A bond pays a dividend of £2 in 4 months. The current price of the bond is 96.4. The yield curve is flat; the continuously compounded interest rate is 5%. What is the arbitrage-free price of a 6-month forward contract on the bond?

B2. How would the answer to B1 change if the yield curve were not flat, but rather you were given zero-coupon bonds prices \( P_t \) (in particular the values for \( t = 4/12 \) and \( t = 6/12 \))? (The answer is a formula involving zero-coupon bond prices.)

B3. Suppose you own a house in Leeds. You are worried that house prices may fall. A financial company offers trading in house price futures contracts. Explain how such futures contracts can be used to hedge your risk, to ease your worries.

B4. Given that the house price futures are based on a UK-wide index/average, is the hedge from B3 likely to be perfect, i.e. to remove all risk? How can this be taken into account when hedging?

QUESTION C

A small bond market consists of 4 bullet bonds. Current time is 0 and payments occur at times 1, 2, 3 and 4. The maturities, coupon rates, and prices (per £100 notional) of the bullet bonds are:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Coupon rate</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>100.97</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>101.94</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>100.21</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>93.49</td>
</tr>
</tbody>
</table>

C1. Show that the (arbitrage-free) zero-coupon bond prices are 0.9709, 0.9246, 0.8641 and 0.7928.

C2. Calculate the 1-year-ahead forward rates, \( f_t \) for \( t = 0, 1, 2, 3 \).

C3. Calculate the yield to maturity of the 3-year bullet bond.

C4. A 2-year annuity bond with a coupon rate of 5% is introduced on the market. Show that the arbitrage-free price (per £100 notional) of the annuity bond is 101.94.

C5. Suppose the annuity bond from C4 trades at the price 101. Explain how to exploit the arbitrage opportunity in the market.

CONTINUED...
QUESTION D

A pension company has to pay out £5m to its policy holders in each of the years 1 to 20 (specifically: at times 1, 2, ..., 20). The yield curve is flat; the annually compounded interest rate is 4%. There is a bond market in which two zero-coupon bonds are traded; one with maturity-date 5 and one with maturity-date 15.

D1. Calculate the present value and the Macauley duration of the company’s liabilities.

D2. How can a(n asset) portfolio of the two zero-coupon bonds be constructed, such the present value and duration of the assets match those of the liabilities?

D3. Does the asset portfolio from D2 fulfill the conditions for Redington immunisation?

QUESTION E

A pension fund invests £30m at time 0. The yearly returns, $i_t$, are independent and identically distributed. The expected value of each $i_t$ is 0.05, each standard deviation is 0.1, and each $\ln(1 + i_t)$ is lognormally distributed.

E1. Calculate the expected value and standard deviation of the accumulated wealth (or asset value; $S_t$) after 15 years.

E2. Calculate the expected value and standard deviation of $\ln(1 + i_t)$.

E3. Calculate the probability that the accumulated wealth ($S_t$) after 15 years is less than £50m. (Attached to this exam paper is a table of the standard normal distribution function.)

END