QUESTION A

A1. State the put-call parity. (You may assume the underlying pays no dividends during the life of the options and that the interest rate is constant.)

A2. A portfolio is long 1 call-option with strike \((K - \epsilon)\), short 2 strike-\(K\) call-options and long one strike-\((K + \epsilon)\) call-option. All the call-options have the same expiry-date, and \(K\) and \(\epsilon\) are positive numbers. Show that this portfolio has positive value — or else there is arbitrage. (Hint: Plot the pay-off profile/function.)
QUESTION B

B1. The £–$ exchange rate is 0.7, i.e. $1 costs £0.7. The continuously compounded interest rate (the force of interest) in the UK is 2% (per year) and 1% in the US. What is the arbitrage-free price of a 6-month forward contract (enabling one to buy $ for £)?

B2. An investor enters into a long position in the forward contract from B1. The next day (1/365th of a year later) the exchange rate is is 0.68. What is the value of the investor’s forward contract?

B3. How will the analysis in B1 and B2 change if “forward” is changed to “futures”?

QUESTION C

A small bond market consists of three bonds. Current time is 0 and payments occur at times 1, 2, and 3. The prices and cash-flows (per £100 notional) are:

<table>
<thead>
<tr>
<th>Price at time 0</th>
<th>Cash-flow at time 1</th>
<th>Cash-flow at time 2</th>
<th>Cash-flow at time 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.97</td>
<td>104</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100.04</td>
<td>4</td>
<td>104</td>
<td>0</td>
</tr>
<tr>
<td>99.43</td>
<td>36.034</td>
<td>36.034</td>
<td>36.034</td>
</tr>
</tbody>
</table>

C1. One of bonds is an annuity. Which one? What is the coupon rate on the annuity bond?

C2. Show that the (arbitrage-free) zero-coupon bond prices are 0.97087, 0.92456, and 0.86384.

C3. Calculate the zero-coupon (spot) rates. Without performing any calculations: How do forward rates compare to zero-coupon (spot) rates?

C4. Find the yield to maturity of the annuity bond. Without performing any calculations: How does the yield to maturity on the annuity bond compare to the 3-year zero-coupon (spot) rate?

C5. Calculate the 2-year par yield.

CONTINUED...
QUESTION D

A pension company has to pay out £5m to its policy holders in each of the years 15 to 19 (specifically: at times 15, 16, ... , 19). The yield curve is flat; the annually compounded interest rate is 4%. There is a bond market in which two bullet bonds are traded; a 5-year bond with coupon rate 3.5% and a 30-year bond with coupon rate 4%. The current time (now) is 0.

D1. Calculate the present value and the Macauley duration of the company’s liabilities.

D2. Calculate the arbitrage-free price (per £100 notional) and the Macauley duration of the two bullet bonds.

D3. How (if it is at all possible) can an asset portfolio of the two bullet bonds be formed such that the combined asset-liability position is immunised to small interest rate changes?

QUESTION E

At each of times 0, 1, and 2 a pension company invests £10m in a particular asset. The yearly returns, \(i_t\), on the asset are independent. In the first year the return will be 8% with probability 1/2, and 2% with probability 1/2. In the second year the return is 2%, 6%, or 10% each with probability 1/3. The return in the third year is 5% with probability 2/3, and 8% with probability 1/3.

E1. Calculate expected the values and standard deviations of (each of) the yearly returns.

E2. Calculate the expected value of the accumulated wealth (or asset value; \(A_t\)) after 3 years.

E3. Give a recursive formula for the expected value of the squared accumulated wealth, \(E(A_t^2)\).

E4. The pension company’s portfolio manager makes the following statement:

\[
\text{At long investment horizons wealth tends to stabilize around its expected value.}
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Comment. (Is the statement true, or false or ... ?)