Exercises for Wednesday October 6 Workshops

Exercise 1.1: When to Replace a Machine
A factory needs a particular machine for its production. Buying a new machine costs £100,000. In its first year, the machine’s maintenance cost (paid at the end of the year) is £10,000, in its second year it is £20,000, third year £30,000, and so on. Every time an old machine is replaced by a new one, this cost pattern repeats itself. The task is now to find the optimal time to replace an old machine, where ”optimal” means that the present value of the total costs (i.e. the cumulative discounted costs) is minimized.
Show that if the machine is replaced every year, the present value of costs (in £1,000’s) solves the equation

\[ PV_{\text{total}}^1 = 100 + \frac{10}{1 + r} + \frac{PV_{\text{total}}^1}{1 + r}, \]

where \( r \) denotes the (yearly compounded) interest rate.
Argue that if machine replacement takes place every \( n \) years, the present value of total costs satisfies

\[ PV_{\text{total}}^n = 100 + \sum_{j=1}^{n} \frac{10j}{(1 + r)^j} + \frac{PV_{\text{total}}^n}{(1 + r)^n}. \]

Calculate the optimal replacement frequency for \( r = 0.1 \) and \( r = 0.05 \).

Exercise 1.2: Currency Forwards
On pages 5-6 Hull gives an argument linking spot (\( S \)) and forward (\( F \)) prices for a stock that does not pay dividends during the life of the forward contract. Written in ”maths notation” it says that

\[ F(t, T) = e^{r(T-t)}S(t), \]

where \( r \) denotes the continuously compounded interest rate.
How does the argument change, if the underlying is instead a currency? Hint: Ask yourself ”I buy US-$ for borrowed money. Do I just hold it, or deposit it in a US bank?” (Recent financial turmoil notwithstanding.) The answer involves the interest rates in the two countries.
Exercise 1.3
Exercise 1.23 from Hull’s Chapter 1.

Exercise 1.4
Exercise 1.25 from Hull’s Chapter 1. Hint: Ask yourself (in case (a) ”What does my payoff after 180 days look like if I buy the call-option and take a short position the forward contract?”