Change of Numeraire in Five Formulas

A numeraire (stock, zcb, bank-account; not so much option):

\[
dS(t) = r(t)S(t)dt + S(t)\sigma_S^T(t) dW^Q(t)
\]

(1)

The \(Q^S\)-Brownian motion:

\[
dW^S(t) = dW^Q(t) - \sigma_S(t)dt
\]

(2)

For any random variable \(Z\):

\[
\beta(t)E^Q_t \left( \frac{Z}{\beta(T)} \right) = S(t)E^S_t \left( \frac{Z}{S(T)} \right)
\]

(3)

With \(\pi\) generic for “price of no-dividend traded asset”, we have;

\[
\pi/S \text{ is a } Q^S\text{-martingale}
\]

(4)

\[
d\pi = (r + \sigma_\pi^T \sigma_S)\pi dt + \pi \sigma_\pi^T dW^S
\]

(5)