Danish zero coupon rates (and some significant events)

![Graph showing Danish zero coupon rates with significant events labeled: Amsterdam Treaty Referendum, Euro Referendum, Russian default, LTCM collapse, Interst Only Mortgages, US Subprime Crisis, Lehman Default, September 11, 1995 to 2010. The x-axis represents the year, and the y-axis represents the interest rate.](image-url)
Besides the *story lines* we see that:

- Interest rates are not constant. (One reason for plural.)

- Interest rates follow some *reasonably stable* stochastic processes (they don’t stray too far from \([0, 0.1]\)) — although the short rate doesn’t look much like a constant-parameter diffusion.

- Long and short rates move somewhat but not perfectly in sync.

- The financial crisis 2007+
  - not really visible until Lehman Bros. default in Sept. 2008
  - levels are still not that extreme (in DK; in US they’re very close to 0); the big effects are in spreads and liquidity.
What determines interest rates? And what moves them around?

- Agents’ preferences for consuming now vs. saving for later. Fear and greed.
- Supply and demand. Not to be forgotten.
- “Usual macroeconomic suspects” such as expected growth rates and inflation, fiscal policy (in the hand of politicians), monetary policy (central banks but subject to political pressure), international effects, exchange rates.
- Institutional structure (labor, housing and mortgage markets, legal and political systems, . . .)
Overload: Too much to model!

Randomness is a very large component. Let’s just accept that and build empirically plausible stochastic models. Worked well for stocks.

Wilmott and Rasmussen quote
Complication: Many assets (bonds) are different (because they pay at different times) but not too different.

That is what interest rate (or term structure or fixed income) modelling is about.

Fixed income markets are huge. DK government bonds at CSX: Around 1,500 billion \(1.5 \times 10^{12}\) DKK. About the same in mortgage-backed bonds. And thats only the securitized stuff. Add bank loans, credit cards, devivatives.

Nice: The martingale formalism, our fundamental theorems of asset pricing (absence of arbitrage, completeness, . . .) carry over.

We will be looking only at non-trivial special cases. (And the more special, the more non-trivial.)
The structure

We skip Björk’s Chapter 22. Model independent results or abstract non-sense, depending on your point of view. One of the best such chapters in the literature, but we want to get to some concrete models.

Chapter 23 is half-way there. Vasicek’s argument. (Works for stochastic volatility too, btw.)

Chapter 24. Vasicek’s model. And Cox-Ingersoll-Ross’. Both affine models. (Called that for a reason.) Lots of stuff we can calculate (dynamics, bond prices, estimates). And concepts we can discuss (market price of risk, calibration).