

Asset Pricing II, Exercise Set #3

Answers to the two exercises below are due at the lectures on March 27-28 2008.

Expected Returns on Calls; In Two Ways

Consider the Black-Scholes model and let - as always - \mathbb{P} denote the physical/real-world/statistical probability measure, and \mathbb{Q} the risk-neutral probabilities or the equivalent martingale measure with the bank-account as numeraire.

Investing until Expiry

Recall that the (simple) rate of return (over the horizon 0 to T) from a call-option investment is

$$R(T) = \frac{(S(T) - K)^+ - \text{Call}(0)}{\text{Call}(0)}.$$

What is $\mathbf{E}^{\mathbb{P}}(R(T))$? And how does this compare to the \mathbb{P} -expected return from a pure stock investment? Does that tell us why somebody might be interested in buying call-options? What is the \mathbb{P} -standard deviation of the option investment return? And how does that compare to that of stock investment? And what about \mathbb{Q} -moments?

Investing; Instantaneous Version

Let us write $\text{Call}(t) = F(t, S(t))$. Show that

$$d\text{Call}(t) = (F_t + \mu S(t)F_S + \frac{1}{2}\sigma^2 S^2(t)F_{SS})dt + \sigma S(t)F_S dW^{\mathbb{P}}.$$

Show that we may rewrite this as

$$d\text{Call}(t) = \text{Call}(t) \left(\underbrace{\left(\frac{(\mu - r)S(t)F_S}{F} + r \right)}_{:=\mu^{\text{Call}}} dt + \frac{\sigma S(t)F_S}{F} dW^{\mathbb{P}} \right),$$

meaning that the instantaneous \mathbb{P} -expected excess rate of return satisfies the CAPM-like equation

$$\mu^{\text{Call}} - r = \frac{S(t)F_S}{F}(\mu - r)$$

Show that for a call-option in the Black-Scholes model

$$\frac{S(t)F_S}{F} > 1.$$

Suppose, quite conceivably, $\mu > r$. What does that tell you about investing in call-options?

What happens if you do the analysis for put-options?

Calibration, Long Rates and Modelling (In)Consistency

Björk's exercise 22.5 with the details of Ho-Lee model calibration. Here you need to do a little reading yourself about calibration.

Björk's exercise 22.7. Björk says that "obviously the limit will depend on $r(t)$ and t ". To me seems more obvious that the limit – if it exists – does *not* depend on $r(t)$ and t . Neither is true, but it can be shown that if the process $f^\infty(t) := \lim_{T \rightarrow \infty} f(t, T)$ is well-defined, then it is increasing. And that there are models – fairly strange ones, though – where the limit depends non-trivially on $r(t)$.