

# Results on Dade's Conjecture

as of March 2001

## 1. SPORADIC SIMPLE GROUPS

$M_{11}$	final	Dade [D1]
$M_{12}$ (+coverings, outerauto.)	final	Dade
$M_{22}$ (+coverings, outerauto.)	final	Huang [H]
$M_{23}, M_{24}$	final	Schwartz, An, Conder [AC]
$J_1$	final	Dade [D1]
$J_2$ (+coverings, outerauto.)	final	Dade
$J_3$ (+coverings, outerauto.)	final	Kotlica [K]
$McL$ (+coverings)	final	Murray [M], Entz, Pahlings [EP]
$Ru$	final	Dade, An, O'Brien [AO3]
$He$	final	An [A5]
$HS$	final	Hassan, Horváth [HH1]
$Co_2$	final	An, O'Brien [AO1]
$Co_3$	final	An [A6]
$Suz$	final	Himstedt [Hi]
$O'Nan$	final	Uno, Yoshiara [UY]
$Fi_{23}$	final	An, O'Brien [AO2]
$Fi_{22}$	invar.	An, O'Brien [AO4]
sporadic, abelian defect principal block	ord.	Rouquier [Rq]

## 2. FINITE CHEVALLEY GROUPS

$GL_n(q)$	ord., $p q$	Olsson, Uno [OU1]
$GU_n(q)$	ord., $p q$	Ku [Ku]
$GL_n(q), GU_n(q)$	invar., $p \nmid q$	An [A10]
$Sp_{2n}(q), SO_m^\pm(q)$	ord., $p \nmid q, p, q$ odd	An [A11]
$L_2(q)$	final	Dade [D5]
$L_3(q)$	final, $p q$	Dade
$L_n(q)$	ord., $p q$	Sukizaki [S]
$Sz(2^{2n+1})$	final	Dade [D5]
$G_2(q)$	final, $p \nmid q, q \neq 3, 4$	An [A1]+
${}^2G_2(3^{2n+1})$	final	$p \neq 3$ An [A2], $p = 3$ Eaton [E1]
${}^2F_4(2^{2n+1})$	ord., $p \neq 2$	An [A3]
${}^2F_4(2)'$ (+ outerauto.)	final	An [A4]

## 3. SYMMETRIC AND ALTERNATING GROUPS

$A_n$ , abelian defect	ord.	Fong, Harris [FH2]
$S_n$	ord.	$p \neq 2$ Olsson, Uno [OU2], $p = 2$ An [A8]

#### 4. GENERAL RESULTS

cyclic defect group case	final	Dade [D4] +
tame block case	invar.	Uno [U]
abel. defect unipotent blocks	ord.	Broué, Malle, Michel [BMM]
abel. defect principal blocks	ord., $p = 2$	Fong, Harris [FH1]
abel. defect, some cases	ord.	Puig, Usami [PUn] [Usn]
$p$ -solvable	proj.	Robinson [Rb1]
$O_p(G)$ cyclic, $G/O_p(G)$ T.I. $p$ -Sylow	proj.	Eaton [E2]

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