3.2.20

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Tesse

And

Works

v. N.

objects: M cld manifolds, aspherical, i.e. $\tilde{M}$ is contractible ($\tilde{M}$ – universal covering)
e.g. $\tilde{\mathbb{T}}^n = \mathbb{R}^n$, so $\mathbb{T}^n$ aspherical

Def: M is called rigid if for every N cld mfld, $M \simeq N$ (htpy equiv), then $M \cong N$ (homeo).

Borel conjecture: Every cld aspherical mfld is rigid

Ex of mflds which are rigid
- dim $= 2$ (uniformization)
- dim $= 3$ (geometrization)
- dim ≥ 5: M has $K \leq 0$ (non-positively curved), or if $\pi_1 M$ is a discrete subgp of $Gl_n(\mathbb{R})$ (Farrel-Jones)
Lück-Bartels: if $\pi_1 M$ is hyperbolic

Def: M is called stably rigid if $\forall N$, $N \simeq M$, there ex homeo $N \times \mathbb{R}^n \cong M \times \mathbb{R}^n$ (for some n)

Thm (Guentner, Tessera, Yu)
M is stably rigid if $\pi_1 M$ belongs to a class $\mathcal{F}$ of ctbl gps,
$\mathcal{F}$ stable under subgps, extensions, amalg. prod,

increasing unions, and F contains all elementary amenable gps, and all subgps of $Gl_n(K)$

pf (idea):

apply „Meyer-Vietoris argument at large scale of $\tilde{M}$", $\tilde{M} = A \cup B$, A, B „nice" open subsets, a notion of being nice should satisfy:

- A bounded open set is nice
- if $A = \coprod_{r\text{-disj}} A_i$ where r is large enough and A_i are nice in a uniform way, then A is nice

$\coprod_{r\text{-disj}} A_i$ means $\coprod A_i$ where $d(A_i, A_j) \geq r$ for $i \neq j$

Finite decomposition complexity (FDC)

Notation: A „metric family" $\mathcal{X} = \{X\}$ is a family of metric spaces.

If $\mathcal{X}, \mathcal{Y}$ are metric families and $r > 0$, we denote $\mathcal{X} \xrightarrow{r} \mathcal{Y}$ if $\forall X \in \mathcal{X}$ $X = X_0 \cup X_1$, $X_i = \coprod_{r\text{-disj}} X_{ij}$ where $\{X_{ij}\} \subset \mathcal{Y}$

Decomposition game: $\mathcal{X}$ metric family

$\mathcal{X}_0 = \mathcal{X}$

$\mathcal{X}$ called bounded if $\sup\{\operatorname{diam}(X) : X \in \mathcal{X}\}$ is finite

player 1	player 2
r_1	$\mathcal{X}_0 \overset{\to}{r_1} \mathcal{X}_1$
r_2	$\mathcal{X}_1 \overset{\to}{r_2} \mathcal{X}_2$
...	...

player 2 wins at step k if $\mathcal{X}_k$ is bdd.

- $\mathcal{X}$ has FDC if player 2 has a winning stategy
- $\mathcal{X}$ has finite ~~decomposition~~ asymptotic dimension (FAD) ~~$\Leftarrow K$~~ if player 2 has a winning strategy in at most K steps for some $K \in \mathbb{N}$.

Ex: $\mathcal{X} = \{\mathbb{R}\}$ has FAD, in one step:

r_1

X_{ij} bdd

countable groups: Γ a ctbl gp, we fix a left-invariant proper metric on Γ

Thm (Guentner, Tessera, Yu)
A cld, aspherical mnfld s.t. its fundam. gp has FDC, is stably rigid.

Ex: $\mathbb{Z}^k$ has FAD

$\bigoplus_{\mathbb{N}} \mathbb{Z}$ does not have FAD, but FDC

For linear gps:

Thm: If Γ is a f.g. subgp of $Gl_n(K)$, then:

- if char $K > 0$, then Γ has FAD
- if char $K = 0$, then Γ has FDC (but possibly not FAD)

Ex: $\mathbb{Z} \wr \mathbb{Z} = \mathbb{Z}^{(\mathbb{Z})} \rtimes \mathbb{Z}$ has FDC but not FAD