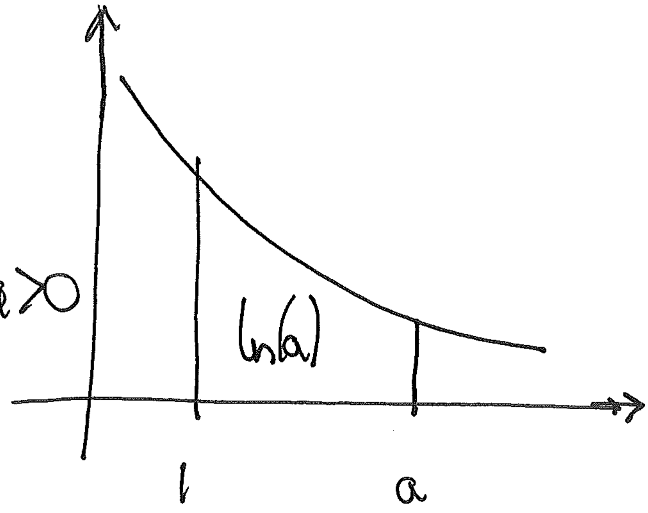
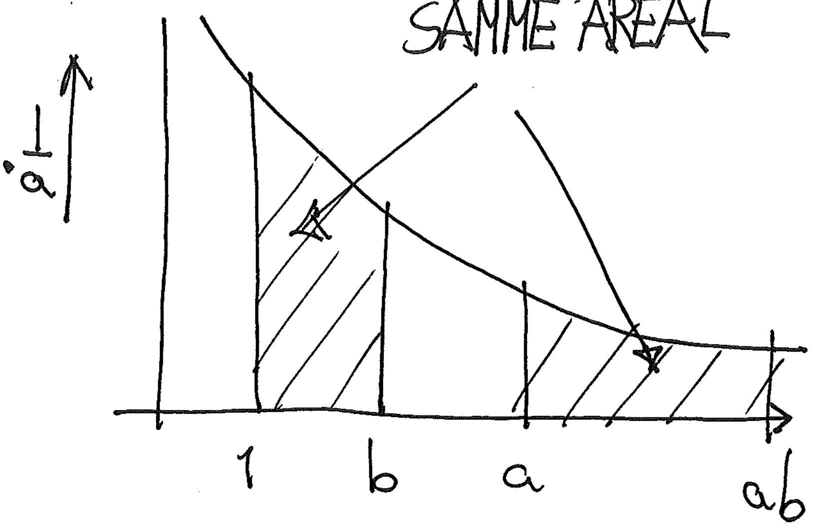
Logaritmefunksjonen

$$\ln(a) = \int_1^a \frac{1}{t}\,dt, \quad a>0$$



$$\ln(a) \to \infty \text{ for } a \to \infty \qquad \ln(a) \to -\infty \text{ for } a \to 0^+$$



$(x,y) \to (ax, \frac{1}{a}y)$ bevarer areal så

$$\int_1^b \frac{1}{t}\,dt = \int_a^{ab} \frac{1}{t}\,dt$$

$$\ln(ab) = \int_1^{ab} \frac{1}{t}\,dt = \int_1^a \frac{1}{t}\,dt + \int_a^{ab} \frac{1}{t}\,dt = \int_1^a \frac{1}{t}\,dt + \int_1^b \frac{1}{t}\,dt$$

$$= \ln(a) + \ln(b)$$

Logaritmefunksjonen sender produkt i sum.

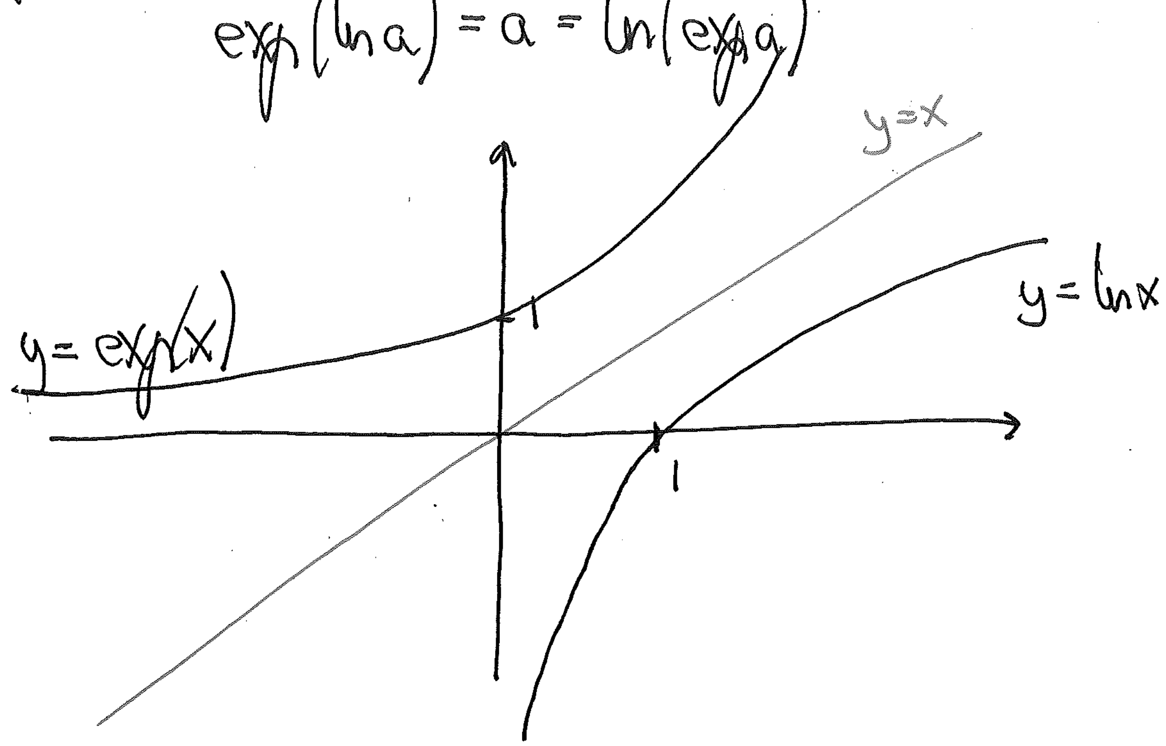
$$\ln(a^b) = \ln(a \cdots a) = \ln(a) + \cdots + \ln(a) = \ln(a)\cdot b$$

Exponential funktionen

$$\mathbb{R}_+ \underset{\exp}{\overset{\ln}{\rightleftarrows}} \mathbb{R}$$

exp er den omvendte funktion til ln:

$$\exp(\ln a) = a = \ln(\exp a)$$



Exponentialfunktionen sender sum i produkt:

$$\exp(a+b) = \exp(a)\cdot\exp(b) \qquad \exp(ab) = \exp(a)^b$$

Hvis vi skriver e for $\exp(1)$:

$$\exp(a) = \exp(1\cdot a) = \exp(1)^a = e^a$$

$$e^{a+b} = e^a e^b \qquad e^{ab} = (e^a)^b$$

Hvad betyder 2^{π} ?

$$2^3 = 2 \cdot 2 \cdot 2$$

$$\left(2^{1/3}\right)^3 = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 2 \text{ så } 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$\left(2^{-3}\right)^{-1} = 2^3 \text{ så } 2^{-3} = \frac{1}{2^3}$$

$$2^{a/b} = \sqrt[b]{2^a} = \sqrt[a]{2^b}$$

$$2^{\pi} = \left(e^{\ln(2)}\right)^{\pi} = e^{\pi \ln(2)}$$