

# Matematik 5GE

## Assignment 1

### Problem 1

Define  $\mathbf{C}P^1$  to be the set of complex 1-dimensional complex subspaces of  $\mathbf{C}^2$ . Show that  $\mathbf{C}P^1$  is a smooth manifold: Find local charts and show that the transition functions are smooth. Have we seen this particular manifold before?

### Problem 2

Show that the map  $\pi: TM \rightarrow M$  is smooth: Find an expression for  $\pi$  in local coordinates.

### Problem 3

- Show that there is a global trivialization  $T\mathbf{R}^n \xrightarrow{\cong} \mathbf{R}^n \times \mathbf{R}^n$ . (Remember the discussion about geometric tangent vectors in  $\mathbf{R}^n$ .)
- Show that there is a global trivialization  $TS^1 \xrightarrow{\cong} S^1 \times \mathbf{R}$ . (Think about why we could find a trivialization for  $\mathbf{R}^n$ .)
- Do you think that there is a global trivialization  $TS^2 \xrightarrow{\cong} S^2 \times \mathbf{R}^2$ ? (No proof required.)

Manifolds  $M$  with global trivializations  $TM \xrightarrow{\cong} M \times \mathbf{R}^n$  are said to be *parallizable*.

### Problem 4

Let  $F: M \rightarrow N$  be a smooth map.

- (1) Define a map  $F_*: TM \rightarrow TN$  such that the diagram

$$\begin{array}{ccc} TM & \xrightarrow{F_*} & TN \\ \pi \downarrow & & \downarrow \pi \\ M & \xrightarrow{F} & N \end{array}$$

commutes.

- (2) Show that  $F_*$  is smooth: Find the expression for  $F_*$  in local coordinates.

(Due: Tuesday March 1st )