January 2007
JMM

Topology

Books, notes, calculators, and computers are allowed at this three hour written exam. You may write your answers in pencil.

Problem 1 (25%)
Let X be a topological space and C a subspace of X. Assume that

- C is closed,
- the subspace topology on C is the discrete topology, and
- C is infinite,

Prove that X is not compact.

Problem 2 (25%)
Let X be a topological space and $\mathcal{B}$ a locally finite closed covering of X. This means that $\mathcal{B}$ is a set of subsets of X such that

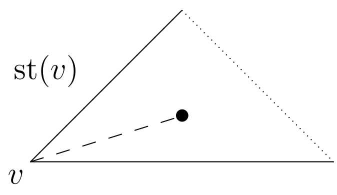
- all sets $B \in \mathcal{B}$ are closed, and
- $X = \bigcup \mathcal{B}$, and
- every point $x \in X$ has a neighborhood U_x such that $\{B \in \mathcal{B} \mid U_x \cap B \neq \emptyset\}$ is finite

Let A be a subset of X. Show that

$$A \cap B \text{ is open in } B \text{ for all } B \in \mathcal{B} \Longrightarrow A \text{ is open in } X$$

Problem 3 (25%)
Let K be an abstract simplicial complex and v a vertex of K.

(1) Show that $K_v = \{\sigma \in K \mid v \notin \sigma\}$ is a subcomplex of K.
(2) Explain why $\mathrm{st}(v) = |K| - |K_v|$ is an open subset of the realization of K.
(3) Prove that $\mathrm{st}(v)$ is path-connected.
(4) Explain why $|K|$ is locally path-connected.



Problem 4 (25%)
Consider the surface presentation $\mathcal{P} = \langle a, b, c, d \mid abdc^{-1}, a^{-1}cd^{-1}b \rangle$. What standard surface is $|\mathcal{P}|$?

(THE END)