CONTAGION VERSUS FLIGHT TO QUALITY IN FINANCIAL MARKETS

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Outline

- Transmission of Risk between Economies
- Definitions of Interdependence and Contagion
- Statistical measures for dependence: Pitfalls of correlation
- Multivariate Extreme Value Theory: A new copula
- Measuring Interdependence and Contagion by tail dependence measures
- Causality in the Extremes
- Application: The flight to quality phenomenon

Transmission of Risk between Economies

Every economy is exposed to a series of factors that can culminate in what can be called crisis.

Types of crises: financial, liquidity, banking or currency crises.

Definition 1. *A general definition of crisis in a market is given by a threshold level such that in case is exceeded, it results in the collapse of the system producing the triggering of negative effects in the rest of the markets.*

In summary: A crisis in one market is characterized by the collapse not only of that market but by the negative effects produced on other markets.

Two ways of regarding dependence: (In particular in crises periods)

From the point of view of the direction (Causality in the Extremes).

From the point of view of the intensity: strength of the links in turmoil periods.

Interdependence and Contagion

- Interdependence due to rational links between the variables (markets).

- Contagion effects : abnormal links between the markets triggered by some phenomena (crisis).

- **Regarding the direction:**

  ⋆ Interdependence implies that both markets collapse because both are influenced by the same factors (Forbes and Rigobon (2001), Corsetti, Pericoli, Sbracia (2002)).

  ⋆ Contagion implies that the collapse in one market produces the fall of the other market.

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  - ⋆ Contagion implies that the collapse in one market produces the fall of the other market.
- **Regarding the intensity:**
  - ⋆ Interdependence implies no significant change in cross market relationships.
  - ⋆ Contagion implies that cross market linkages are stronger after a shock to one market.

Transmission Channels connecting the markets

From an economic viewpoint:

• Economic fundamentals, market specific shocks, impact of bad news, phycological effects (herd behavior).

From an statistical viewpoint: Pearson correlation.

$$Corr(X_1, X_2) = \frac{E(X_1 - E(X_1))(X_2 - E(X_2))}{\sqrt{V(X_1)}\sqrt{V(X_2)}},$$

with X_1 and X_2 random variables.

Correlation is not sufficient to measure the dependence found in financial markets.

- It is only reliable when the random variables are jointly gaussian.
- Conditioning on extreme events can lead to misleading results.

Pitfalls of Correlation

These results are found in Embrechts, et al. (1999) and in Boyer et al. (1999).

- Correlation is an scalar measure (Not designed for the complete structure of dependence).
- A correlation of zero does not indicate independence between the variables.
- Correlation is not invariant under transformations of the risks.
- Correlation is only defined when the variances of the corresponding variables are finite.
- An increase in the correlation between two variables can be **JUST** due to an increase in the variance of one variable.

  Ex.- Let ρ be the correlation between two $r.v.$'s X, Y and let us condition on $X \in A$. Then $\rho_A = \rho \left(\rho^2 + (1-\rho^2) \frac{V(X)}{V(X|A)} \right)^{-1/2}$

SOLUTION: A complete picture of the structure of dependence (Copula functions).

Copula functions for dependence

Definition 2. *A function* $C : [0,1]^m \to [0,1]$ *is a* m*-dimensional copula if it satisfies the following properties:*

(i) For all $u_i \in [0,1]$, $C(1,\ldots,1,u_i,1,\ldots,1) = u_i$.

(ii) For all $u \in [0,1]^m$, $C(u_1,\ldots,u_m) = 0$ *if at least one of the coordinates is zero.*

(iii) The volume of every box contained in $[0,1]^m$ *is non-negative, i.e.,* $V_C([u_1,\ldots,u_m] \times [v_1,\ldots,v_m])$ *is non-negative. For* $m = 2$, $V_C([u_1,u_2] \times [v_1,v_2]) = C(u_2,v_2) - C(u_1,v_2) - C(u_2,v_1) + C(u_1,v_1) \geq 0$ *for* $0 \leq u_i, v_i \leq 1$.

By Sklar's theorem (1959),

$$H(x_1,\ldots,x_m) = C(F_1(x_1),\ldots,F_m(x_m)),$$

with H the multivariate distribution, and F_i the margins.

Our Goal: Using dependence in the Extremes

Let $(M_{n1}, \ldots, M_{nm})$ be the vector of maxima, and denote its distribution by

$$H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = P\{a_{ni}^{-1}(M_{ni} - b_{ni}) \leq x_i, i = 1, \ldots, m\}.$$

The central result of EVT in the multivariate setting ($mevt$) is:

$$\lim_{n\to\infty} H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = G(x_1, \ldots, x_m),$$

with G a $mevd$.

Theorem 1. *The class of $mevd$ is precisely the class of max-stable distributions (Resnick (1987), proposition 5.9).*

These distributions satisfy the following Invariance Property,

$$G^t(tx_1, \ldots, tx_m) = G(\alpha_1 x_1 + \beta_1, \ldots, \alpha_m x_m + \beta_m),$$

for every $t > 0$, and some $\alpha_j > 0$ and β_j.

By Sklar's theorem,

$$\lim_{n\to\infty} H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = C(G_1(x_1), \ldots, G_m(x_m)),$$

with G_i univariate evd.

Under an appropriate transformation of the margins ($Z_i = 1/log\frac{1}{F_i(X)}$),

$$\lim_{n\to\infty} H^{*n}(nz_1, \ldots, nz_m) = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)), \tag{1}$$

with $\Psi_1(z) = \exp(-\frac{1}{z})$, standard Fréchet, and the invariance property for copulas reads

$$C^n(\Psi_1(nz_1), \ldots, \Psi_1(nz_m)) = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)).$$

Taking logs in both sides of (1) and applying the invariance property we have

$$\lim_{n\to\infty} \frac{H^*(nz_1, \ldots, nz_m)}{1 + log\, C(\Psi_1(nz_1), \ldots, \Psi_1(nz_m))} = 1.$$

Then, $H^*(z_1, \dots, z_m) = C(\Psi_1(z_1), \dots, \Psi_1(z_m))$, from some threshold vector $(z_1, \dots, z_m)$ sufficiently high.

- The copula function C is derived from the limiting distribution of the maximum.
- C must be of exponential type (extension of the EVT for the univariate case).

The Gumbel copula is within this class. Its general expression is

$$C_G(u_1, \dots, u_m; \theta) = \exp^{-[(-log\ u_1)^{\theta} + \dots + (-log\ u_m)^{\theta}]^{1/\theta}}, \quad \theta \geq 1,$$

with $u_1, \dots, u_m \in [0, 1]$ and $\theta \geq 1$.

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Our aim: Modelling the complete structure of dependence between the variables. Not just the relation in the extremes!

Our Contribution: A NEW Copula

WE PROPOSE instead (for m=2):

$$\widetilde{C}_G(u_1, u_2; \Theta) = \exp^{-D(u_1,u_2;\gamma,\eta)[(-log\ u_1)^\theta + (-log\ u_2)^\theta]^{1/\theta}}, \tag{2}$$

with

$$D(u_1, u_2; \gamma, \eta) = \exp^{\gamma(1-u_1)(1-u_2)^\eta}, \quad \gamma \geq 0, \quad \eta > 0. \tag{3}$$

The function $D(u_1, u_2; \gamma, \eta)$ accommodates departures from the invariance property with $\gamma > 0$ and $\eta \neq 1$.

Theorem 2. *The function* $\widetilde{C}_G : [0,1] \times [0,1] \to [0,1]$ *defined in* (2) *and* (3) *is a copula function if the parameters in* Θ *satisfy that* $\widetilde{c}_G(u_1, u_2; \Theta) > 0$, $\forall\ (u_1, u_2) \in [0,1] \times [0,1]$, *with* $\widetilde{c}_G(u_1, u_2; \Theta) = \frac{\delta^2 \widetilde{C}_G(u_1,u_2;\Theta)}{du_1 du_2}$ *the density function of the copula* $\widetilde{C}_G$.

Advantages of this NEW Copula

- This copula function is derived from the multivariate extreme value theory, in contrast to ad-hoc models for the dependence structure.

- The function $D(u_1, u_2; \gamma, \eta)$ and in particular the parameter γ extend the multivariate extreme value theory results to the entire range of the random variables.

- $\widetilde{C}_G$ is able to explain asymmetric effects of the variables for $\eta \neq 1$, and $\gamma > 0$.

- This copula is sufficiently flexible to describe different forms of dependence,
  - ⋆ Dependence: $\theta \neq 1$ or $\theta = 1$ and $\gamma > 0$.
  - ⋆ Independence: $\gamma = 0$, $\theta = 1$.
  - ⋆ Asymptotic dependence: $\theta > 1$.
  - ⋆ Asymptotic independence: $\theta = 1$.

Our Contribution: Tail Dependence Measures

- Alternatives to the standard $\aleph$,

$$\aleph = \lim_{t \to \infty} P\{Z_2 > t | Z_1 > t\},$$

introduced by Ledford and Tawn (1997) and Coles, Heffernan and Tawn (1999).

- Definitions of Interdependence and Contagion by means of tail dependence measures.

- The translation of these definitions to mathematical expressions by using copula functions.

- The distinction between types of contagion: In Intensity and In the direction.

Interdependence

Lehman (1966) defined two random variables Z_1, Z_2 as positively quadrant dependent (PQD) if for all $(z_1, z_2) \in \mathbb{R}^2$,

$$P\{Z_1 > z_1, Z_2 > z_2\} \geq P\{Z_1 > z_1\}P\{Z_2 > z_2\},$$

or equivalently if

$$P\{Z_1 \leq z_1, Z_2 \leq z_2\} \geq P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}.$$

Definition 3. *Two random variables are Interdependent if they are PQD. Interdependence is characterized by joint movements in the same direction (co-movements).*

In terms of the copula Interdependence amounts to see that $g(u_1, u_2) > 0$, with

$$g(u_1, u_2) = \widetilde{C}_G(u_1, u_2) - u_1 u_2.$$

Contagion in Intensity

A stronger condition is required: **Tail Monotonicity**.

Definition 4. *Suppose Z_1, Z_2 with common Ψ_1 and consider z a threshold that determines the extremes in the upper tail of both random variables. There exists a contagion effect between Z_1 and Z_2 if $g(u_1, u_2)$ is an increasing function for both random variables, and for $u_1, u_2 \geq u$ with $u = \Psi_1(z)$.*

For the lower tails contagion in intensity is characterized by decreasing tail monotonicity in

$P\{Z_1 \leq z_1, Z_2 \leq z_2\} - P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}$.

In terms of copulas contagion in the upper tails amounts to

$$h_1(u_1, u_2) = \frac{\delta \widetilde{C}_G(u_1, u_2)}{du_1} - u_2 > 0, \qquad h_2(u_1, u_2) = \frac{\delta \widetilde{C}_G(u_1, u_2)}{du_2} - u_1 > 0.$$

Directional Contagion: Causality in the Extremes

The conditional probability is interpreted as a causality relationship.

Let z be a threshold determining the extremes for both random variables.

Motivation: $P\{Z_2 > z'|Z_1 > z\} > P\{Z_2 > z'\} \stackrel{?}{\equiv} Z_1 \Rightarrow Z_2$, with $z' > z$.

(Z_1 taking on extreme values is causing that Z_2 takes on extreme values).

However, This is not true!

False Intuition:

$$P\{Z_2 > z'|Z_1 > z\} > P\{Z_2 > z'\} \equiv P\{Z_2 > z', Z_1 > z\} > P\{Z_2 > z'\}P\{Z_1 > z\}$$

This condition determines Contagion in Intensity NOT in the direction (No causality).

Assuming a common marginal $d.f.$ Ψ_1, and a threshold z determining the extremes for both random variables, we find contagion spill-over from Z_1 to Z_2 if

$$P\{Z_2 > z' | Z_1 > z\} > P\{Z_1 > z' | Z_2 > z\},$$

or equivalently if

$$P\{Z_2 \leq z' | Z_1 \leq z\} > P\{Z_1 \leq z' | Z_2 \leq z\}.$$

These conditions boil down to see

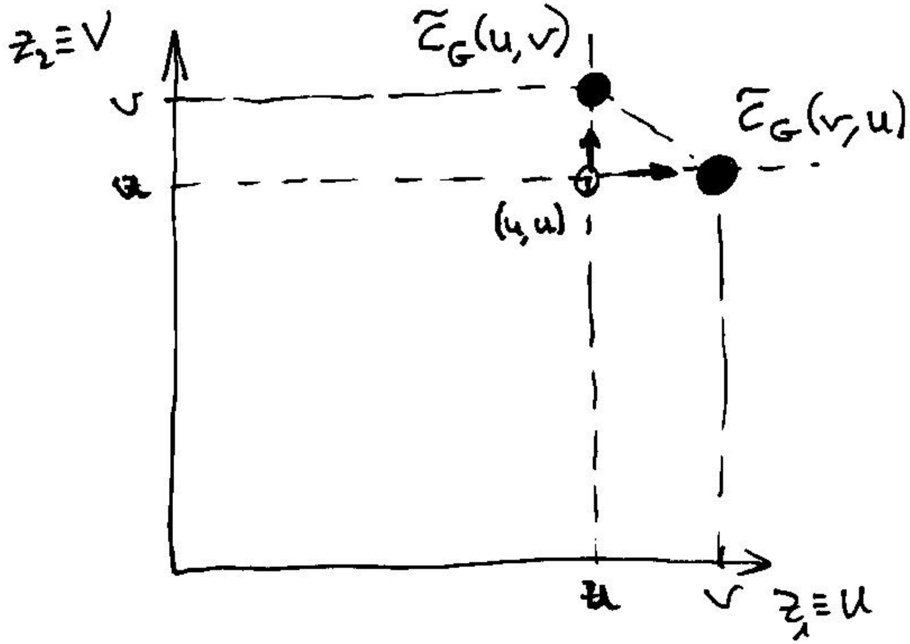
$$\widetilde{C}_G(u,v) > \widetilde{C}_G(v,u) \quad for \quad Z_1 \Rightarrow Z_2 \quad (\text{Causality in the extremes}),$$

with $u = \Psi_1(z)$, and $v = \Psi_1(z')$.

Define $gd_v(u) = \widetilde{C}_G(u,v) - \widetilde{C}_G(v,u)$. Then

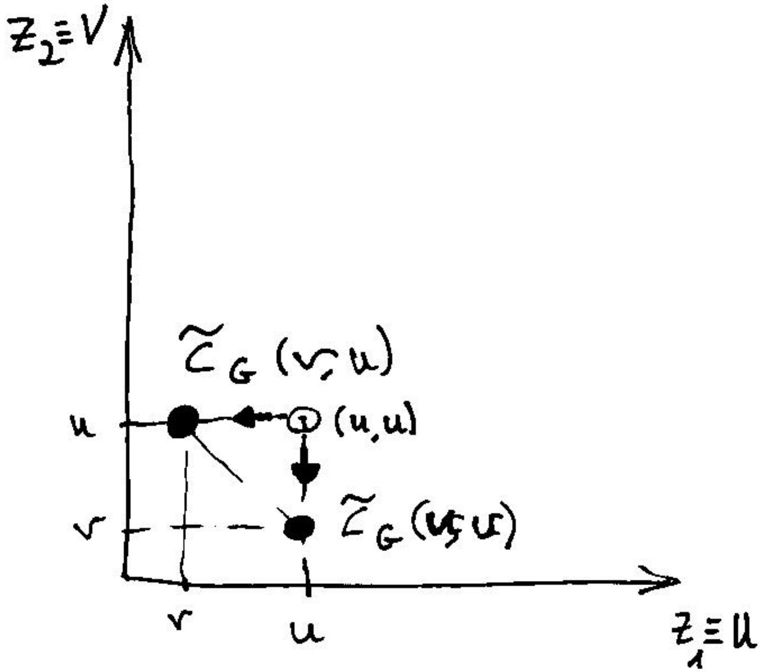
Definition 5. *Z_1 is influencing Z_2 in the extreme values (contagion effect) if $gd_v(u)$ is strictly positive for all $v > u$ for the upper tail, and for all $v < u$ for the lower tail, with $u = \Psi_1(z)$.*

Intuition



$\tilde{C}_G(u,v) > \tilde{C}_G(v,u) \Rightarrow Z_1 \not\Rightarrow Z_2$

(In the upper tail)



$\tilde{C}_G(u,v) > \tilde{C}_G(v,u) \Rightarrow Z_1 \Rightarrow Z_2$

(In the lower tail)

Application: Flight to quality versus Contagion

Definition 6. *Outflows of capital from the stock markets (Z_2) to the bonds markets (Z_1) in crises periods.*

This is represented by

$$P\{Z_1 > z | Z_2 < z'\} - P\{Z_1 > z\} > 0,$$

with z defining the extreme values in the upper tail, and z' in the lower tail.

Experiment: Dow Jones Corporate 02 Years Bond Index (DJBI02) vs Dow Jones Industrial Average: Dow 30 Industrial Stock Price Index (DJSI).

General Model:

$$\left.\begin{array}{l} X_{1,t} = g_1(X_{1,t-1}, X_{2,t-1}) + \varepsilon_{1,t} \\ X_{2,t} = g_2(X_{1,t-1}, X_{2,t-1}) + \varepsilon_{2,t} \end{array}\right\}$$

with $(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim \widetilde{C}_G$.

Financial Sequence: $X_{i,t} = 100\,(logP_{i,t} - logP_{i,t-1})$, $i = 1, 2$, with $P_{i,t}$ the corresponding prices.

Modelling Rational Dependence

DJBI02 index is well modelled by an AR(1)-GARCH(1,1) model as follows,

$$X_{1,t} = 0.00025 + 0.089 X_{1,t-1} + \sigma_{1,t}\varepsilon_{1,t}, \text{ with } \varepsilon_{1,t} \ i.i.d. \ (0,1),$$

and $\sigma^2_{1,t} = 6.194 \cdot 10^{-8} + 0.071 \varepsilon^2_{1,t-1} + 0.903 \sigma^2_{1,t-1}$.

DJSI Index is modelled by the following pure GARCH(1,1) model,

$$X_{2,t} = \sigma_{2,t}\varepsilon_{2,t}, \text{ with } \varepsilon_{2,t} \ i.i.d. \ (0,1),$$

and $\sigma^2_{2,t} = 3.0012 \cdot 10^{-6} + 0.096 \varepsilon^2_{2,t-1} + 0.887 \sigma^2_{2,t-1}$.

The evolution of prices in one market is independent of the other.

The irrational dependence (dependence in the innovations) is measured by the links between the vectors $(\varepsilon_{1,t}, \varepsilon_{2,t})$ and $\widetilde{C}_G$.

Estimate of $\widetilde{C}_G$: $\hat{\theta}_n = 1.031$, $\hat{\eta}_n = 1$ and $\hat{\gamma}_n = 0.175$. $(\Downarrow)$

IGARCH Effect

Consider

$$X_t = \sigma_t \varepsilon_t, \text{ with } \varepsilon_t \ i.i.d. \ (0,1),$$

and

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

with $\alpha + \beta = 1$.

Features of the model:

- $V(X_t) = \infty$.
- In the same way that $I(1)$ represents **persistence** in linear models, $IGARCH(1,1)$ describes **persistence** in the square and absolute observations.
- Persistence, **NOT Long Range Dependence**, because the latter implies finite marginal variances.

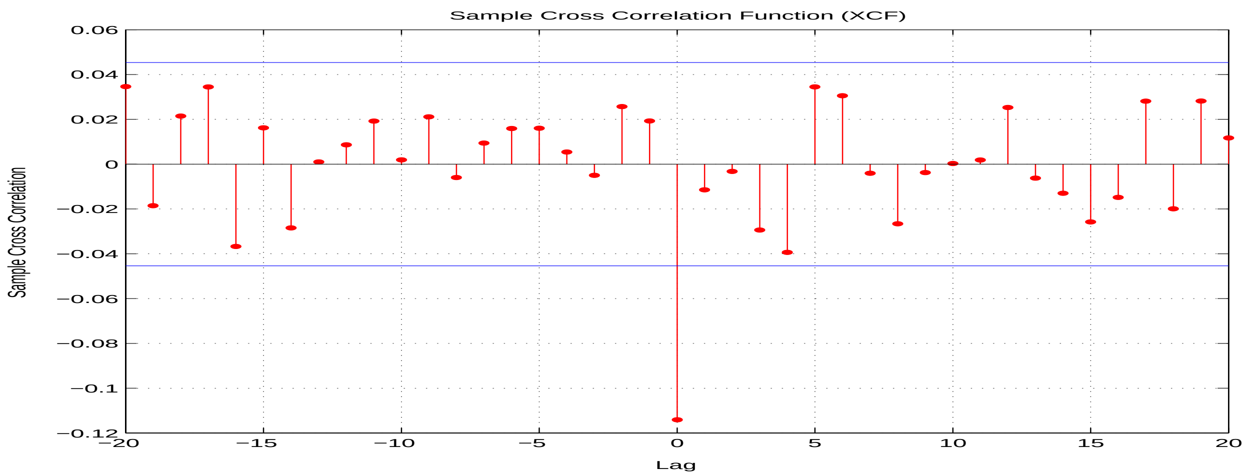
However, the IGARCH effect may show up by (Mikosch and $St\breve{a}ric\breve{a}$):

- Persistence in the squares (true IGARCH).
- Non-stationarity due to different regimes (different means, different stationary GARCH, etc.)

Regarding Contagion:

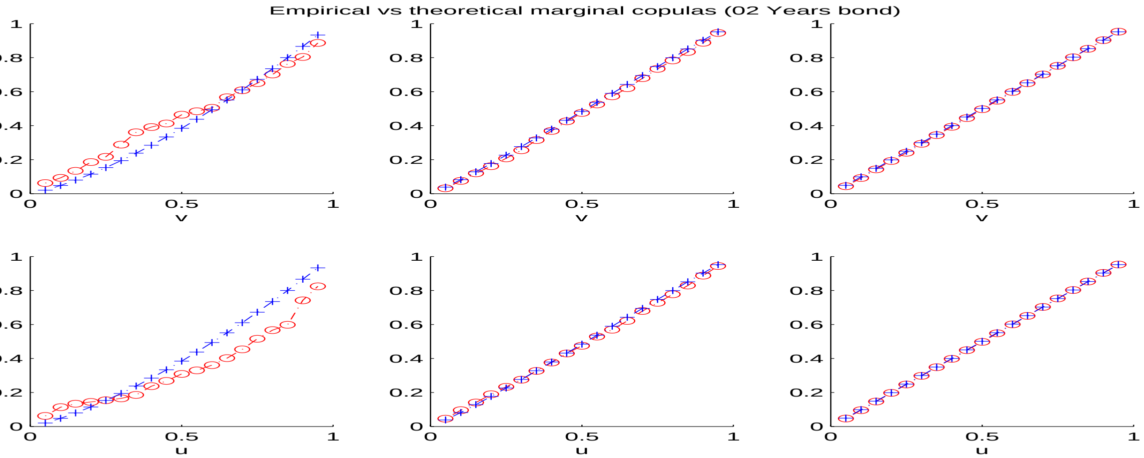
- For true IGARCH: Study the contagion effect for the vector of innovations $(\varepsilon_1, \varepsilon_2)$ obtained from the IGARCH model.
- Non-stationarity: Consider the univariate sequence $\{X_t\}$ and filter it by the corresponding regimes to obtain a sequence of innovations ε_t that is $I(0)$ and serially independent.

Modelling Irrational Dependence



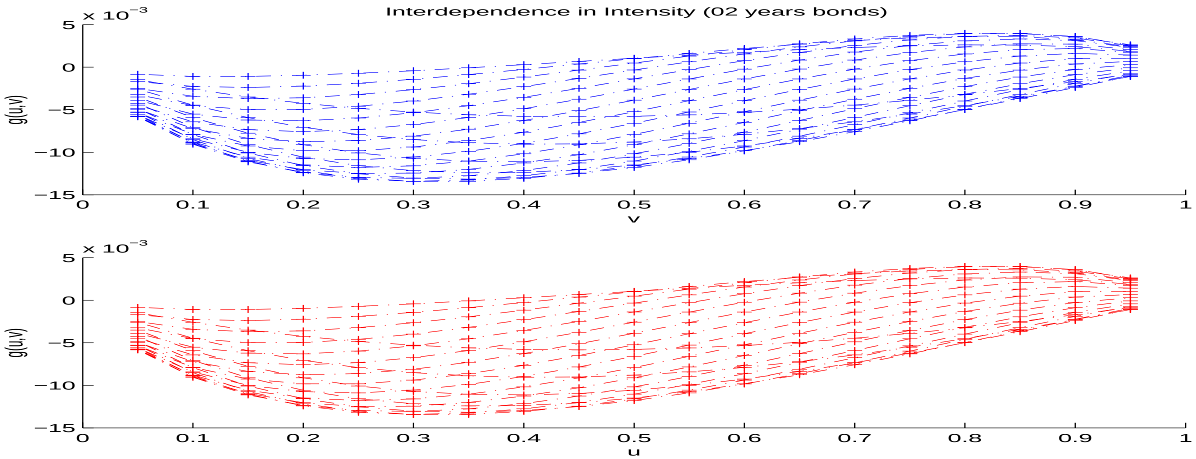
Cross correlation for different lags of the bivariate innovation sequence, spanning the period $02/01/1997 - 24/09/2004$, $n = 1942$ observations.

Goodness of Fit Test



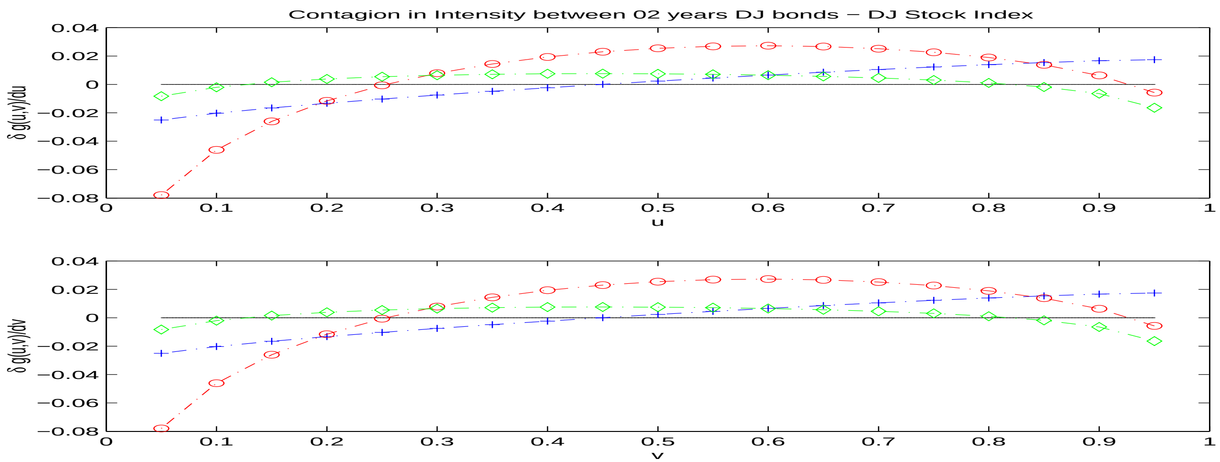
Empirical ($o-$) and theoretical ($+-$) margins. The upper panel for the vertical sections and the lower panel for the horizontal section. 0.05 quantile, 0.50 quantile and 0.95 quantile respectively.

Interdependence in Intensity



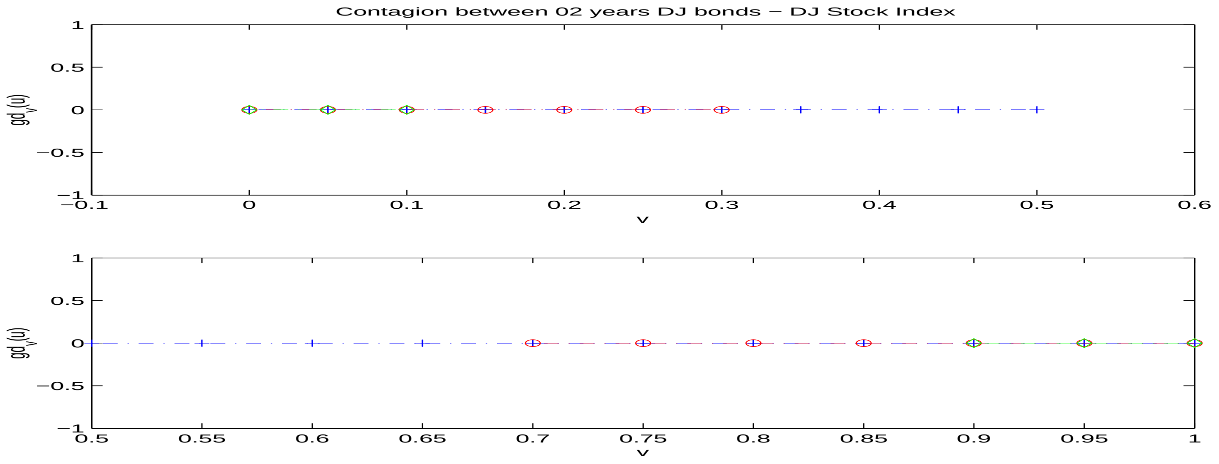
The upper panel depicts the function $g(u, v)$ plotted against the innovations of $DJSI$. The lower panel $g(u, v)$ plotted against the innovations of $DJBI02$.

Contagion in Intensity



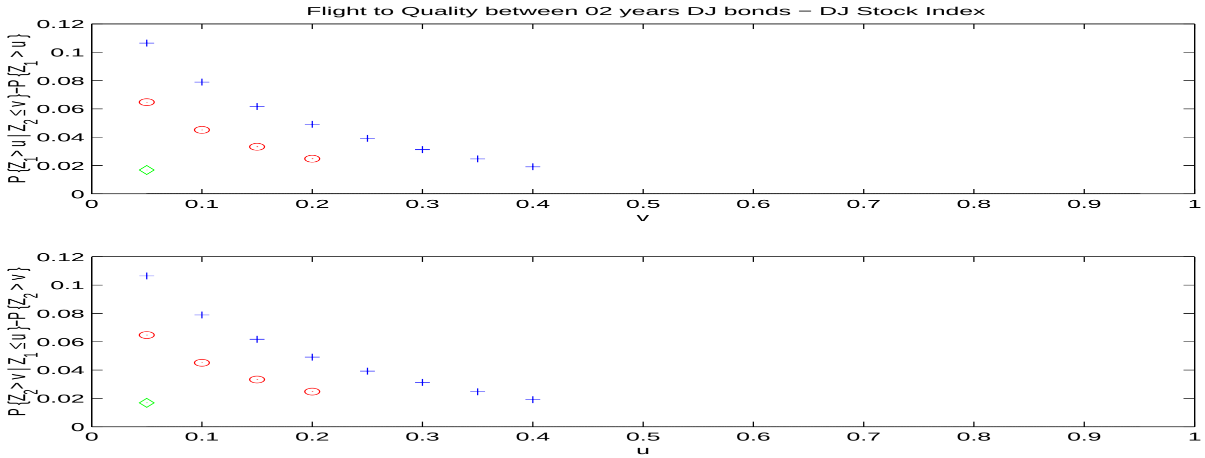
The upper panel depicts $h_1(u,v)$ against $DJBI02$ and the lower panel depicts $h_2(u,v)$ against $DJSI$. $(+-)$ for 0.05 quantile, $(o-)$ the 0.50 quantile and $(\diamond-)$ the 0.95 quantile.

Directional Contagion



The upper panel depicts $gd_v(u)$ for $v \leq u$. $(+-)$ for $u = 0.50$, $(o-)$ for $u = 0.30$ and $(\diamond-)$ for $u = 0.10$. The lower panel depicts $gd_v(u)$ for $v > u$. $(+-)$ for the $u = 0.50$, $(o-)$ for $u = 0.70$ and $(\diamond-)$ for $u = 0.90$.

Flight to Quality: $P\{Z_1 > u | Z_2 < v\} - P\{Z_1 > u\} > 0$



In the upper panel $(+-)$ for $u = 0.60$, $(o-)$ for $u = 0.80$ and $(\diamond-)$ for $u = 0.95$. In the lower panel $(+-)$ for $v = 0.60$, $(o-)$ for $v = 0.80$ and $(\diamond-)$ for $v = 0.95$.

Some Interesting Facts

- Negative interdependence in the left tail, that turns positive in the right tail.
- Absence of directional contagion (Symmetric effects between the variables).
- Strong opposite movements in the middle of the domain (negative interdependence) that decrease when the variables take on extreme values. (Intensity Contagion without Interdependence).
- Evidence of Flight to Quality in both tails. This phenomenon can be interpreted as a substitution effect between bonds ($DJBI02$) and stocks ($DJSI$) when either of the sequences are in crises periods.
- DJBI02 depends on its past and in the volatility dynamics.
- DJSI depends only on its volatility dynamics.