Space time analysis of extreme values

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Summary Points

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- Model fitting via customized Markov chain Monte Carlo (MCMC) methods.
- Extreme values of ozone levels in Mexico City.
- Extreme values of rainfall in Venezuela.
Figure 1: Daily maximum values of ozone levels.
Extreme Value Modeling
The traditional approach is based on the *Generalized Extreme Value* (GEV) distribution function:

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H(z) = \exp \left\{ - \left[ 1 + \xi \left( \frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}
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- \(-\infty < \mu < \infty; \sigma > 0; -\infty < \xi < \infty.\)
- \(+\) denotes the positive part of the argument.
- \(\xi > 0\) Fréchet family; \(\xi < 0\) the Weibull family;
  \(\xi \rightarrow 0\) Gumbel family.
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• These ideas had been developed into a Bayesian hierarchical modeling framework. Smith et al. (1997); Assuncao et al. (2004); Casson and Coles (1999); Gilleland et al. (2004).
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• Deterministic functions: \( \mu_t = \beta_0 + \beta_1 t; \mu_t = \beta_0 + \beta_1 + \beta_2 t + \beta_3 t^2 \) or \( \mu_t = \beta_0 + \beta_1 X_t \).
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- Non-stationarity can also be included for the shape and/or scale parameters: $\sigma_t = exp(\beta_0 + \beta_1 t); \xi_t = \beta_0 + \beta_1 t$ or $\xi_t = \beta_0 + \beta_1 t + \beta_2 t^2$. 
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- We propose the use of Dynamic Linear Models (DLM) as in West and Harrison (1997) to model the parameter changes in time.
GEV distribution with DLM’s
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- For $z_1, z_2, \ldots, z_m, z_t \sim GEV(\mu_t, \sigma, \xi)$

$$H_t(z_t) = \exp \left\{ -[1 + \xi(z_t - \mu_t)/\sigma]_{+}^{-1/\xi} \right\}$$

$$\mu_t = \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V)$$

$$\theta_t = \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V)$$
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- Parameters \((t = 0)\) are assumed apriori independent.

- \( \pi(\sigma) \sim LN(m_\sigma, s_\sigma) \); \( \pi(\xi) \sim N(m_\xi, s_\xi) \).

- \( \theta_0 \sim N(m_0, C_0) \); \( V \sim IG(\alpha_v, \beta_v) \); \( \tau \sim IG(\alpha_\tau, \beta_\tau) \).
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- $\theta_0 \sim N(m_0, C_0); V \sim IG(\alpha_v, \beta_v); \tau \sim IG(\alpha_\tau, \beta_\tau)$.

- $\mu_t$ follows a first order polynomial DLM with state vector $\theta_t$. 
General DLM \((F_t, V, G_t, W)\)
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\[
\begin{align*}
\mu_t &= F_t' \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\
\theta_t &= G_t \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, W)
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- \( \theta_t \) is a \( k \times 1 \) state vector;
- \( F_t \) is a \( k \times 1 \) regressor vector;
- \( G_t \) is a \( k \times k \) evolution matrix;
- \( V \) is an observational variance and
- \( W \) is a \( k \times k \) evolution covariance matrix.
Posterior Inference for DLM-GEV models
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- Define \( Z = (z_1, z_2, \ldots, z_m) \); \( \mu = (\mu_1, \mu_2, \ldots, \mu_m) \) and \( \theta = (\theta_1, \theta_2, \ldots, \theta_m) \).
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- $p(\mu_t | z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.
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- $p(\mu_t|z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.

- $p(\sigma|Z, \mu, \xi)$ and $p(\xi|Z, \mu, \sigma)$ are also sampled via M-H.
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- $V$ and $W$ are sampled from Inverse Gamma/Wishart distributions.
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• For $\theta_t$, we apply Forward Filtering Backward Simulation (FFBS) as in Carter and Kohn or Frühwirth-Schnatter (1994).
Forward in time, we obtain $p(\theta_t | D_t, V, W); t = 1, 2, \ldots, m$
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- Backwards in time, we sample \( p(\theta_m|D_m, V, W) \) and then, recursively we sample from \( p(\theta_t|\theta_{t+1}, D_m, V, W); t = m - 1, \ldots, 1 \)
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- Dynamics for scale/shape parameters. Sequential updating with *Particle Filters*. 
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– Observation variance equals zero.
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  – Dynamics for scale/shape parameters. Sequential updating with *Particle Filters*.

  – Observation variance equals zero.

  – No space or space-time structure.
Figure 2: Posterior mean for $\mu_t$, and 90% probability interval for $\theta_t$; 1990-2002 data
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- Is there an overall decreasing trend in the maxima of the previous figure?
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- Alternatively, we considered a regression model on $\mu_t$ with time-varying intercept but a constant slope:
Detecting Trends

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• One possibility is to add an extra model parameter and estimate incremental growth.

• Alternatively, we considered a \textit{regression} model on $\mu_t$ with time-varying intercept but a constant slope:

$$ z_t \sim GEV(\mu_t, \sigma, \xi) $$
$$ \mu_t = \theta_t + \beta(t - \bar{t}) + \epsilon_t; \quad \epsilon_t \sim N(0, V) $$
$$ \theta_t = \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V) $$
where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and $\beta$ represents change of level per unit of time.
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- For model fitting, notice that:
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- For model fitting, notice that:
  - Conditional on \( \theta_t \), the difference \( \mu_t - \theta_t \) follows a regression model.
  - Conditional on \( \beta \), \( \mu_t - \beta(t - \bar{t}) \) follows a first order polynomial DLM.
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• For model fitting, notice that:
  
  – Conditional on $\theta_t$, the difference $\mu_t - \theta_t$ follows a regression model.
  
  – Conditional on $\beta$, $\mu_t - \beta(t - \bar{t})$ follows a first order polynomial DLM.
  
  – This defines a *Gibbs sampler* scheme that produces posterior samples for $\beta$ and $\theta_t$. 
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  - This defines a *Gibbs sampler* scheme that produces posterior samples for $\beta$ and $\theta_t$.

- In fact, $Pr(\beta < 0|Z) \approx 0.79$ indication of a decreasing trend.
Figure 3: Posterior distribution for $\beta$; ozone data 1990-2002.
Maxima monthly rainfall values in Venezuela
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• The model is:

$$Y_t \sim GEV(\mu_t, \sigma, \xi)$$

$$\mu_t = \theta_t + \beta_t X_t + \epsilon_t$$

$$\theta_t = \theta_{t-1} + \omega_{1t}$$

$$\beta_t = \beta_{t-1} + \omega_{2t}$$
Figure 4: Maxima monthly rainfall values in Venezuela and NAO index

(a) Time


0 40 80 120

(b) Time


−4 −2 0 2 4

NAO index

Time

Rainfall
Figure 5: Posterior median and 90% probability intervals for $\beta_t$
Space-time model with Process Convolutions
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- Consider data $y_t = (y_{1,t}, \ldots, y_{n_t,t})'$ which is recorded at sites $s_1, \ldots, s_{n_t}$. 
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- A possible model (Higdon 2002) is
  
  \[
  y_t = K^t x_t + \epsilon_t \\
  x_t = x_t + \nu_t
  \]
Space-time model with Process Convolutions

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- A possible model (Higdon 2002) is

$$y_t = K^t x_t + \epsilon_t$$
$$x_t = x_t + \nu_t$$

$K^t$ is a $n_t \times \kappa$ matrix given by

$$K^t_{ij} = k(s_i - \omega_j), \ t = 1, \ldots, m$$

$$\epsilon_t \sim N(0, \sigma^2_{\epsilon}), \ t = 1, \ldots, m$$

$$\nu_t \sim N(0, \sigma^2_{\nu}), \ t = 1, \ldots, m$$

$$x_1 \sim N(0, \sigma^2_x I_{\kappa})$$
- $k(\cdot - \omega_j)$ defines a smoothing kernel.

- $\omega_1, \ldots, \omega_\kappa$ are spatial sites where kernels are centered.

- $x_t$ is interpreted as a **latent process**

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y(s, t) = \sum_j k(s - \omega_j)x_{jt}
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• $k(\cdot - \omega_j)$ defines a smoothing kernel.

• $\omega_1, \ldots, \omega_\kappa$ are spatial sites where kernels are centered.

• $x_t$ is interpreted as a latent process

$$y(s, t) = \sum_j k(s - \omega_j)x_{jt}$$

• Possible kernels are:
  
  – **Gaussian**: $k(s) \propto \exp \left\{ -\|s\|^2/2\eta \right\}$ ($\eta > 0$).
  
  – **Exponential**: $k(s) \propto \exp \left\{ -\|s\|/\eta \right\}$ ($\eta > 0$).
  
  – **Spherical**: $k(s) \propto \left( 1 - \frac{\|s\|^3}{r^3} \right)^3 I[s \leq r]$. 
A Spatio-Temporal Model for the GEV distribution
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- Assume $y_{s,t} \sim GEV(\mu_{s,t}, \sigma, \xi); s = 1, \ldots, S, t = 1, \ldots, m$

$$H_{s,t}(y_{s,t}; \mu_{s,t}, \xi, \sigma) = \exp \left\{ - \left[ 1 + \xi \left( \frac{y_{s,t} - \mu_{s,t}}{\sigma} \right) \right]^{-1/\xi} \right\}$$
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\]

- For each \( t, \mu_t = (\mu_{1,t}, \mu_{2,t}, \ldots, \mu_{S,t})' \). \((\sigma, \xi)\) constant in time.
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- For each $t$, $\mu_t = (\mu_{1,t}, \mu_{2,t}, \ldots, \mu_{S,t})'$. $(\sigma, \xi)$ constant in time.

- We define a DLM on $\mu_t$:

$$\mu_t = K'\theta_t + \epsilon_t;$$

$$\theta_t = \theta_{t-1} + \nu_t$$
• $\theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})'$, $\epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})'$, $\nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'$. 
\[ \theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})', \ \epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})', \ \nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'. \]

\[ \epsilon_t \sim N(0, \sigma^2_\epsilon I_{\kappa \times \kappa}); \ \nu_t \sim N(0, \sigma^2_\nu I_{\kappa \times \kappa}) \]
• \( \theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})' \), \( \epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})' \), \( \nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})' \).

• \( \epsilon_t \sim N(0, \sigma^2_\epsilon I_{\kappa \times \kappa}) \); \( \nu_t \sim N(0, \sigma^2_\nu I_{\kappa \times \kappa}) \)

• With a Gaussian kernel, \( K' \) is an \( S \times \kappa \) matrix with entries:

\[
K'_{ij} = K(s_i - \omega_j);
\]

\[
K(s_i - \omega_j) \propto \exp(-d||s_i - \omega_j||^2/2)
\]

• \( s_i \) is the position of station \( i \).

• \( \omega_j \) is the position of the kernel \( j = 1, \ldots, \kappa \).
\( \theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})', \epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})', \nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})' \).

\( \epsilon_t \sim N(0, \sigma_\epsilon^2 I_{\kappa \times \kappa}); \nu_t \sim N(0, \sigma_\nu^2 I_{\kappa \times \kappa}) \)

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| \( K(s_i - \omega_j) \propto \exp(-d||s_i - \omega_j||^2/2) \) |

- \( s_i \) is the position of station \( i \).
- \( \omega_j \) is the position of the kernel \( j = 1, \ldots, \kappa \).
- \( d \) is a range parameter; \( d = c\phi; 1/2 < c < 2; \phi = \text{knot distance} \).
• 1st stage priors: \( \pi(\sigma) \sim LN(\mu_\sigma, s_\sigma) \) and \( \pi(\xi) \sim N(\mu_\xi, s_\xi) \) are th
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• 2nd stage priors: $\theta_0 \sim N(0, \sigma_\theta^2 I_{K\times K})$; $1/\sigma_\epsilon^2 \sim Gamma(\alpha_\epsilon, \beta_\epsilon)$; $1/\sigma_\nu^2 \sim Gamma(\alpha_\nu, \beta_\nu)$ and $1/\sigma_\theta^2 \sim Gamma(\alpha_\theta, \beta_\theta)$. 
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• The log-likelihood is equal to

$$l(\theta) = -mS \log \sigma - \sum_{t=1}^{m} \sum_{s=1}^{S} \left[ 1 + \xi \left( \frac{z_{s,t} - \mu_{s,t}}{\sigma} \right) \right]^{-1/\xi} +$$

$$- \left( 1 + \frac{1}{\sigma} \right) \sum_{t=1}^{m} \sum_{s=1}^{S} \log \left[ 1 + \xi \left( \frac{y_{s,t} - \mu_{s,t}}{\sigma} \right) \right] +$$
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- The full conditionals of $\sigma_c^2$; $\sigma_\nu^2$ and $\sigma_\theta^2$ are sampled with Inverse Gamma distributions.
- The range parameter $d$ is assumed fixed, $d = c\phi$. 
Figure 6: RAMA stations, kernel and interpolation grid positions.
Figure 7: Daily maxima for 1999 and posterior estimates of 0.5 quantile.

AZC

XAL

TPN

TAX
Figure 8: Posterior estimate of the $0.5$ quantile of the space-time GEV distribution for a $50 \times 50$ resolution grid.
Figure 9: $u_t = G(y_t)$ diagnostics based on leaving one station out
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