ESTIMATION OF SOME AEOLIAN SALTATION TRANSPORT PARAMETERS FROM TRANSPORT RATE PROFILES

Michael Sørensen
University of Aarhus
Denmark

Abstract

The method for analysing observed aeolian sand transport profiles presented in Jensen and Sørensen (1985) is developed and investigated further. The method involves a mathematical model of aeolian saltation. Detailed information about the saltation process can be calculated from the transport rate profiles by means of the model. Wind tunnel experiments as well as field results are analysed. Estimates are obtained of the mean jump length, the mean impact angle and the probability distribution of the grain speed during saltation at particular heights. These results are found to be in fair accordance with results obtained by direct observation. Also the dislodgement rate, the size distribution of the saltating sand, the grain concentration profile and the grain borne shear stress are calculated. Finally, a method of calculating the wind velocity profile in the saltation layer is proposed.

1. Introduction

One of the most difficult problems in the investigation of aeolian sand transport is to observe what goes on in the layer of dense saltation close to the bed. Our knowledge about the interaction between the sand bed and the saltating grains is poor, but such knowledge is essential for the development of an integrated mathematical model of the entire process of aeolian sand transport. In particular, information is needed about the launch velocity vector of the saltating grains and about the dislodgement rate, i.e. the flux of grains from the bed into the air.

More or less direct observations of launch velocities were obtained by Ellwood et al. (1975), White and Schulz (1977), Nalpanis (1985) and Willetts and Rice (1985). In the three last-mentioned papers grain trajectories were recorded photographically. When this method is used in a fully developed saltation cloud, the low trajectories are obscured by other grains at most wind speeds. Thus White and Schulz were not able to observe trajectories lower than 0.5 - 0.75 cm. The other authors tried to overcome this difficulty in various ways. Ellwood et al. shot grains into a sand layer in still air, while Nalpanis did his experiments at wind speeds just above threshold. Willetts and Rice reduced the saltation by partitioning off a section of their wind tunnel, where sand was fed in just in front of a narrow band of sand. It is not, however, a matter of course that the direct observations of launch velocities obtained so far are typical of well developed saltation at usual wind speeds.

In the present paper an alternative approach is used that can supplement the direct observations. The idea of this method, which is a development of earlier work (Jensen and Sørensen, 1983 and 1985), is to estimate the dislodgement rate and the probability distribution of the launch velocity vector from observed transport rate profiles by means of a mathematical model of aeolian saltation. This is done for each grain size separately.

From a statistical point of view the launch velocity distribution is over-parametrized in this paper. This means that the observations could be explained equally well by more than one probability distribution. However, it is possible to find a reasonable estimate of the probability distribution. Using this estimate and the saltation model some aeolian saltation transport parameters are calculated and, in the few cases where it is possible, compared to direct observations. These comparisons are the background for calling the obtained estimate reasonable.

A way of making the estimation problem more regular in a statistical sense is to postulate that the launch velocity distribution belongs to a class of probability distributions parametrized by only a few parameters. This paper should partly be seen as a way of getting a feel for what class of probability distributions it would be reasonable to use. Also results mentioned above by Nalpanis, White and Schulz and Willetts and Rice will be helpful for the choice of an appropriate class of probability distributions. In the saltation model it would also be possible to supplement the observed transport rate profiles by other kinds of observations when estimating the dislodgement rate and the launch velocity distribution. Possible such observations are data on jump lengths or grain speeds in the air.

The plan of the paper is as follows. In Section 2 the saltation model is presented, while the details about the data are given in Section 3. Here the estimation procedure is explained too. The estimated probability distribution of the launch velocity vector is discussed in Section 4, while the mean jump length, the mean impact angle and the velocity of the saltating grains are considered in Section 5. Section 6 is concerned with the dislodgement rate, and in Section 7 a way of calculating the wind velocity profile in the saltation layer is presented.

2. The saltation model

A number of mathematical saltation models have been proposed in the literature: Bagnold (1936), Ford (1957), Owen (1964,1980), Ellwood, Evans and Wilson (1975), White et al. (1976), White and Schulz (1977), Reizes (1978), Salaün-Penquer, Nassar and Guillaume (1983), Jensen and Sørensen (1983,1985) and Rumpel (1985). The model used in the present paper is, apart from assumptions about the launch speed, identical to the model used in Jensen and Sørensen (1985). In accordance with many other models, the motion of the grains is assumed two-dimensional, and the lift force on a grain is ignored when the grain has left the sand bed. This does not exclude the possibility that fluid lift plays a role while the grains are being launched. It simply means that there is no lift force term in the equations of motion that govern the trajectory of a sand grain when it has been started. These equations are

$$\begin{aligned} \ddot{x} &= H(w)(U(y) - \dot{x}) \\ \ddot{y} + g + H(w)\dot{y} &= 0, \end{aligned} \qquad (2.1)$$

where x is the horizontal distance from the starting point of the trajectory to the present position of the grain, and y is the altitude of the grain. Moreover, g is the acceleration of gravity, $U(y)$ is the time-averaged wind speed at height y and w is the relative velocity between the grain and the air. The function $H(w)$ is given by

$$H(w) = D(w)/mv,$$

where $D(w)$ is the drag exerted on the grain at the relative velocity w, and m denotes the mass of the grain. A more comprehensive discussion of equations (2.1) can be found in Jensen and Sørensen (1983). Note, that the possible effect of turbulent eddies on the motion of the grains is not taken into account.

As proposed by Bagnold (1936), $H(w)$ is calculated as if the sand grain was a sphere with the same specific gravity and the same terminal velocity of fall in air as the grain itself. This is the so-called aerodynamically equivalent sphere. The drag on a sphere is found by the formula

$$D(w) = \frac{\pi}{8} \rho \, c(ws/\nu) w^2 s^2,$$

where ρ and ν are respectively the density and the kinematic viscosity of the air, and s is the diameter of the sphere. The quantity $R = ws/\nu$ is the Reynold number. The drag coefficient $c(R)$ for a sphere is calculated by the formula given by Schiller and Nauman (1933)

$$c(R) = (24/R)(1 + 0.15R^{0.687}).$$

This formula is a good approximation when R is less than 700, which is always the case for aeolian saltation.

The time-averaged wind profile is approximated by the expression

$$U(y) = \begin{cases} 2.5U_* \ln(y/y_0) & \text{if} \quad y \geq \bar{y} \\ 2.5U_* \ln(\bar{y}/y_0)\sqrt{y/\bar{y}} & \text{if} \quad y < \bar{y}, \end{cases} \tag{2.2}$$

where U_* is the friction velocity, and y_0 is the level at which the wind speed would have been zero had the logarithmic profile not been replaced by a square root profile at height $\bar{y}$. That the wind profile over a sand bed with saltating grains is logarithmic at a distance from the bed is well-established (Bagnold, 1941, Owen, 1964). Measurements indicate that the wind speeds in the layer of dense saltation are larger and more uniform than predicted by a logarithmic profile. A sufficient number of reliable measurements of wind speeds in the saltation layer has not yet been obtained, so the formula used close to the bed is only an informed guess. Owen (1964) derived theoretically an asymptotic expansion for the wind profile in the saltation layer based on a number of assumptions. The first term of this expansion was used to calculate the wind speeds in the saltation layer by Owen (1980) and Jensen and Sørensen (1983). This corresponds to assuming that the wind speed is constant in the saltation layer, which we now believe exaggerates the effect of the grains on the wind. A square root profile seems to us more in accordance with what has been measured in the saltation layer. Based on Figure 2 in Owen (1964) we have chosen the value 1 cm for $\bar{y}$. Our results depend only slightly on this choice. We shall return to the wind speed profile in Section 7.

So far the model has been based on basic physics and on what is known about saltation. The only thing missing at this point is the launch velocity vector. We will let this vector be a stochastic variable, whose probability distribution we will estimate from observed transport rate profiles. This probability distribution is dependent on the grain size. We have not made any assumptions about what happens at the bed. In particular, the mechanism that lifts the grains away from the bed is not specified. Most of the grains are presumably rebounding, some are ejected after being hit by an incoming grain, and some are started by fluid lift. Combinations of these mechanisms are quite possible.

There is little doubt that a lift force acts on a saltating grain very close to the bed. This force stems from the narrow gap between the grain and the bed, the steep wind profile gradient, and the rotation of the grain. Presumably the lift force is negligible far from the bed, and we will think of the lift as a part of the launching mechanism. The estimation procedure used in this paper will, in accordance with this interpretation, include any effect of a lift force in the probability distribution of the launch speed. It would, of course, be desirable to include the effect of fluid lift in the equations of motion. However, at present our knowledge about fluid lift on saltating sand grains is much too scarce to do so in a reliable way. Since the observed transport rate profiles studied in this paper can be explained without a lift force term in the equations of motion it seems sensible to avoid this complication.

It should be realized that the model described above does not take account of the rippling of the sand bed. The model must be interpreted as an average of the saltation process over a number of ripples. Obviously, it would be interesting to model the interdependence of the transport process with such phenomena as the creation of ripples and temporal or spatial inhomogenities causing grain sorting. However, we believe that we must be able to handle the simplest kind of aeolian sand transport before we can proceed to the more complicated situations.

3. The data and the method of estimation

Transport rate profiles can be obtained by means of a trap with chambers at a number of heights. The sand caught in each chamber can be divided into size classes by sieving. In this way the transport rate into each chamber can be determined for each individual size class. The trap, of course, disturbs the wind flow and hence also to some extent affects the paths of the saltating grains. This problem will not be discussed in the present paper.

In this paper we will analyse saltation profile data from a field investigation at a beach near Hanstholm on the Danish west coast, from a wind tunnel experiment carried out at Aarhus University and from wind tunnel experiments reported in Williams (1964). The trap used at Hanstholm was 9.8 cm high and was divided into 7 chambers, each with an opening area of 3.5 cm^2. The trap was ventilated through holes along the sides covered by a very fine mesh. The sand at the beach had a typical diameter of 300 μm and the friction velocity was 65 cm/s. Details of this field investigation can be found in Jensen et al. (1984) and Rasmussen et al. (1985). The trap used in the Aarhus wind tunnel experiment was 10.8 cm high and was divided into 12 chambers, each with an opening of 7.9 cm^2. The lowest chamber was open in both ends in order to allow the wind to pass through and not erode the sand in front of the trap. Otherwise, the trap was not ventilated. The transport rate at the height of the lowest chamber was found by extrapolation from the heights above. In this experiment the friction velocity was 46 cm/s and the typical grain size was

229 μm. The size distribution of the sand in the bed is plotted in Fig.6.2. Further details about this wind tunnel experiment will be given in a later paper. Williams' wind tunnel experiments were carried out at three wind speeds with friction velocities 57 cm/s, 83 cm/s and 128 cm/s. The typical grain size was 300 μm. The size distribution of the bed material is plotted in Fig.6.2. The trap used had 1 cm high chambers at five heights only, the lowest between 0 cm and 1 cm above the bed, the highest between 15 cm and 16 cm. Transport rates at other heights were found by interpolation. Here, only a part of Williams' experiment is considered. A comprehensive analysis of the entire experiment can be found in Jensen and Sørensen (1985).

Examples of the data used are shown in Fig.3.1. Here the transport rate profiles obtained in the Aarhus wind tunnel for grains in the ranges 106 - 129 μm and 211 - 258 μm are plotted on a logarithmic scale. The observed profile is less regular for the small grains (106 - 129 μm) than for the grains close to the mode (211 - 258 μm), in particular far from the bed. This is as one would expect. The transport rate of the entire sand population very accurately decreases exponentially with height. Near the bed the transport profiles of the individual size classes are also well described by an exponential decrease, but this is not the case far from the bed. Therefore, an exponential decrease fitted to the 6 lowest observations was used to estimate the transport rate into the lowest chamber, which was open in both ends.

Let $p(\vec{v}_I|s)$ denote the probability density of the launch velocity vector $\vec{v}_I$ for grains of size s, and let $\phi(s)$ denote the dislodgement rate of sand of size s. The dislodgement rate

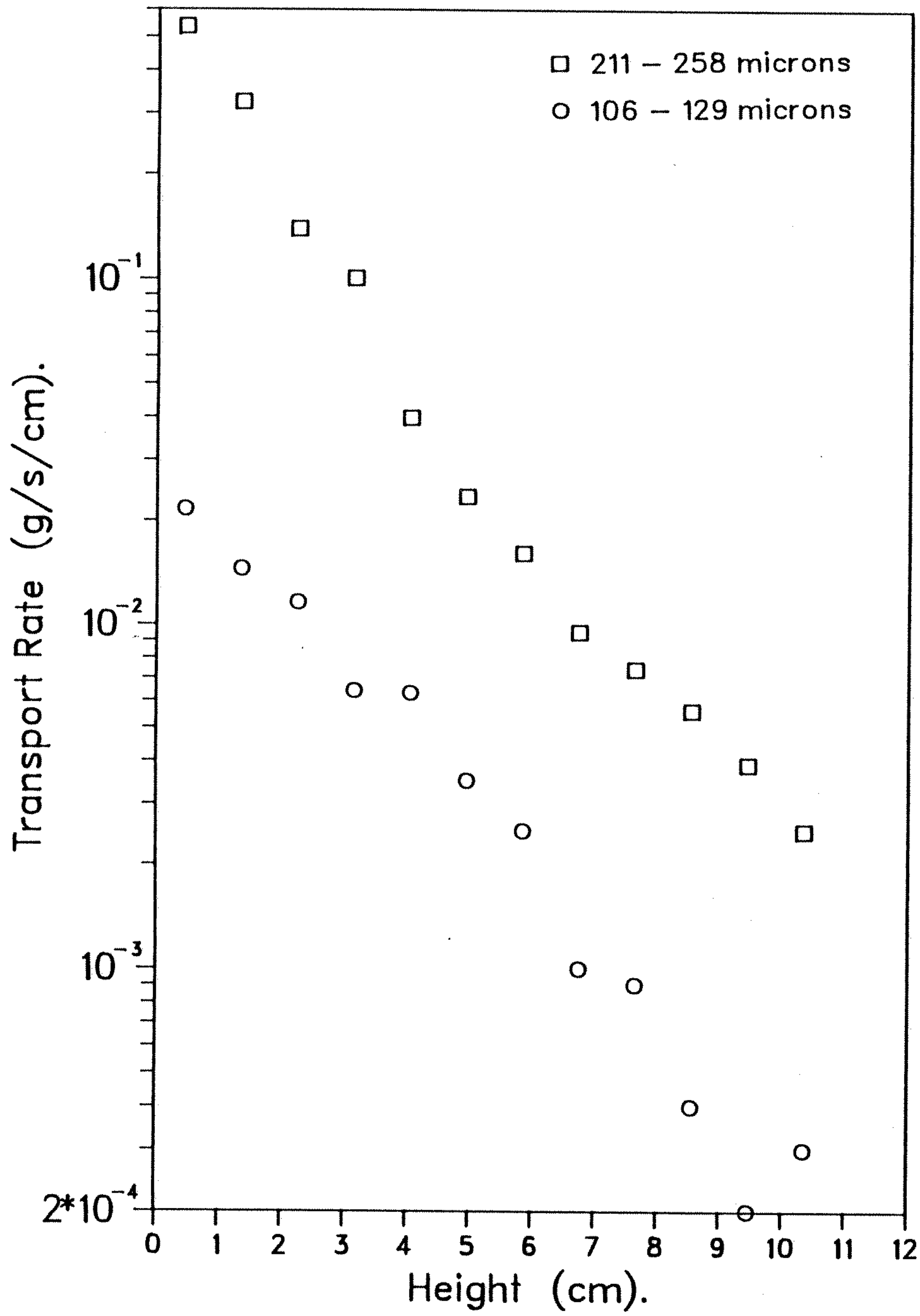


Fig.3.1. Transport rate profiles for the size class 106 - 129 µm and 211 - 258 µm obtained in the Aarhus wind tunnel experiment.

is the mass flux from the bed into the air. It is supposed that p and ϕ do not depend on time and position on the surface, i.e. the saltation is assumed homogeneous and stationary. The mass of grains of size s and initial velocity $\vec{v}_I$ that starts per second is $\phi(s)p(\vec{v}_I|s)d\vec{v}_I$. By (u_1,v_1) and (u_2,v_2) we will denote the velocity vector of a grain when the vertical component of the velocity is respectively positive and negative. The time a grain spends between the heights y_1 and y_2 $(y_1 < y_2)$ on its way up is

$$\int_{y_1}^{y_2} v_1(y)^{-1}dy,$$

and the time spent between y_2 and y_1 on the way down is

$$-\int_{y_1}^{y_2} v_2(y)^{-1}dy.$$

Therefore, the concentration by mass, i.e. the mass per unit volume, at height y of grains on their way up of size s and with launch velocity $\vec{v}_I$ is

$$\phi(s)p(\vec{v}_I|s)v_1(y,\vec{v}_I,s)^{-1}d\vec{v}_I, \tag{3.1}$$

while the concentration of such grains on their way down is

$$-\phi(s)p(\vec{v}_I|s)v_2(y,\vec{v}_I,s)^{-1}d\vec{v}_I. \tag{3.2}$$

Here, of course, y is less than the height of the trajectory of a grain of size s with launch velocity $\vec{v}_I$. We will denote this height by $y^*(\vec{v}_I)$. In analogy with Owen (1964) these expressions for the grain concentration could also have been obtained by a continuity argument.

Suppose that the i'th trap extends from y_1 to y_2 $(y_1 < y_2)$, and let V_{y_1} denote the set of values of $\vec{v}_I$ for which a grain of size s will jump higher than y_1. Moreover, let $y_2 \wedge y^*$ be the minimum of y_2 and y^*. Then the mass c_i of grains caught per second in the i'th trap is given by

$$c_i = \phi(s) \int_{V_{y_1}} \int_{y_1}^{y_2 \wedge y^*(\vec{v}_I)} \left[\frac{u_1(y,\vec{v}_I,s)}{v_1(y,\vec{v}_I,s)} - \frac{u_2(y,\vec{v}_I,s)}{v_2(y,\vec{v}_I,s)}\right] p(\vec{v}_I|s)\,dy\,d\vec{v}_I$$

$$= \phi(s) \int_{V_{y_1}} \Delta x_i(\vec{v}_I,s)\,p(\vec{v}_I|s)\,d\vec{v}_I$$

$$= \phi(s)\overline{\Delta x}_i(s), \qquad (3.3)$$

where $\Delta x_i(\vec{v}_I,s)$ is the horizontal distance travelled by a grain of size s while it passes the i'th chamber (up as well as down) provided its launch velocity is $\vec{v}_I$. The quantity $\overline{\Delta x}_i(s)$ is the mean value of $\Delta x_i(\vec{v}_I,s)$ with respect to the probability distribution $p(\vec{v}_I|s)$. Note, that in particular this shows that the transport rate of grains of size s is the product of $\phi(s)$ and the mean jump length for grains of size s.

In order to calculate the mean value $\overline{\Delta x}_i(s)$ we have to discretise the distribution p. We will allow the launch velocity to take N values $\vec{v}_I^{(j)}$, $j = 1,\ldots,N$. The probability that the value $\vec{v}_I^{(j)}$ is taken is p_j. Then by (3.3) $\underline{c} = (c_1,\ldots,c_n)$ is given by

$$\underline{c} = \phi(s)\underline{\underline{D}}\,\underline{p}\,, \tag{3.4}$$

where n is the number of trap chambers, $\underline{p} = (p_1,\ldots,p_N)$, and where the $n \times N$-matrix $\underline{\underline{D}} = \{d_{ij}\}$ is given by

$$d_{ij} = \Delta x_i(v_I^{(j)},s).$$

Let $T_1,\ldots,T_n$ denote the masses caught per second and per unit trap width in the trap chambers during an experiment. The probabilities $p_1,\ldots,p_N$ and the dislodgement rate $\phi(s)$ can in more than one way be chosen so that $c_i = T_i$ for all i. This is obvious from (3.4). Two such choices of $\underline{p}$ were used in the papers Jensen and Sørensen (1983,1985). In these papers restrictive assumptions were made about the launch angle, and the grains were not allowed to jump over the trap. In the present paper the only restrictions on the launch velocity are that the launch angle is smaller than 90° and that the length of the launch velocity vector is less than the wind speed at height 25 cm.

In order to investigate how well a potential launch velocity distribution $\underline{p}$ explains the observations $T_1,\ldots,T_n$, we calculate the probabilities

$$q_i = c_i/c_{\cdot}\,,$$

where $c_. = \sum_i c_i$. The quantity q_i is the probability that a saltating grain hits the i'th chamber given that it hits the trap. Note, that the q's are independent of $\phi(s)$. We will compare the probabilities q_i to their empirical equivalents

$$r_i = T_i/T_.,$$

where

$$T_. = \sum_i T_i.$$

The comparison is done by means of the Kullback-Leibler information of the empirical probabilities $\underline{r}$ with respect to the distribution $\underline{q}(\underline{p})$, i.e.

$$K(\underline{p}) = \sum_i r_i \ln(r_i/q_i(\underline{p})).$$

The Kullback-Leibler information is a kind of distance measure. In particular, it is non-negative and it is zero if and only if $q_i(\underline{p}) = r_i$ for all i.

Now, we will seek an estimate of $\underline{p}$ by a procedure that aims at making the individual probabilities p_i as equal as possible under the condition that $\underline{p}$ shall explain the data well. Initially all p_i's are given the same value. Then each p_i is changed in turn by a small amount δ or $-\delta$ and all p's are divided by $1+\delta$ or $1-\delta$. The quantity δ is fixed and the sign of δ is chosen so that $K(\underline{p})$ decreases. If either sign increase $K(\underline{p})$ nothing is done. This procedure is first carried out for all p's corresponding to the smallest possible angle and then through all the possible angles in turn to the largest. In the next series of adjustments of the p's the angles are taken in reversed order. After many such iterations the values of $\underline{p}$ stabilize at values where K is close to zero. We can then make $K(\underline{p})$ even smaller by repeating the procedure using a smaller δ. This procedure converged in all cases where it was tried out.

The fact that the statistical problem is over-parametrized implies that we cannot expect our estimate of the probability density p to be smooth. A more regular density can be obtained by applying a smoothing technique, see e.g. Titterington (1985). This has, however, not been done in this paper because the marginal distributions of primary interest turned out to be sufficiently smooth for our purposes.

The idea behind the estimation procedure used in this paper is closely related to the idea of maximum entropy estimation (see e.g. Kotz and Johnson, 1985), and presumably our estimate of $\underline{p}$ approximates the maximum entropy estimate. This point will be investigated in the future.

When $\underline{p}$ has been estimated, an estimate of the dislodgement rate $\phi(s)$ can be obtained from the equations $c_i = T_i$, $i = 1,\ldots,n$, and (3.4).

For grains with a diameter less than 90 μm it turned out that the model predicted that when the launch speed was less than the assumed upper limit, such grains could not reach the upper trap chambers. Actually, grains less than 90 μm were caught in these chambers, so this clearly demonstrates that the model cannot describe the motion of these very fine grains. Presumably their trajectories are affected by the turbulent eddied, cf. Nalpanis (1985). The grains smaller than 90 μm constitute only a tiny fraction of the grain population, and in the rest of this paper we will confine our interest to grains larger than 90 μm.

4. The launch velocity

In this section we shall study the estimated probability distributions of the launch velocity vector. We will concentrate on the results from the Aarhus wind tunnel experiment.

In Fig.4.1 is plotted the estimated probability density function for the length of the launch velocity vector in the case of the size class around the mode point (211 μm - 258 μm). The plot indicates that this probability density might be well

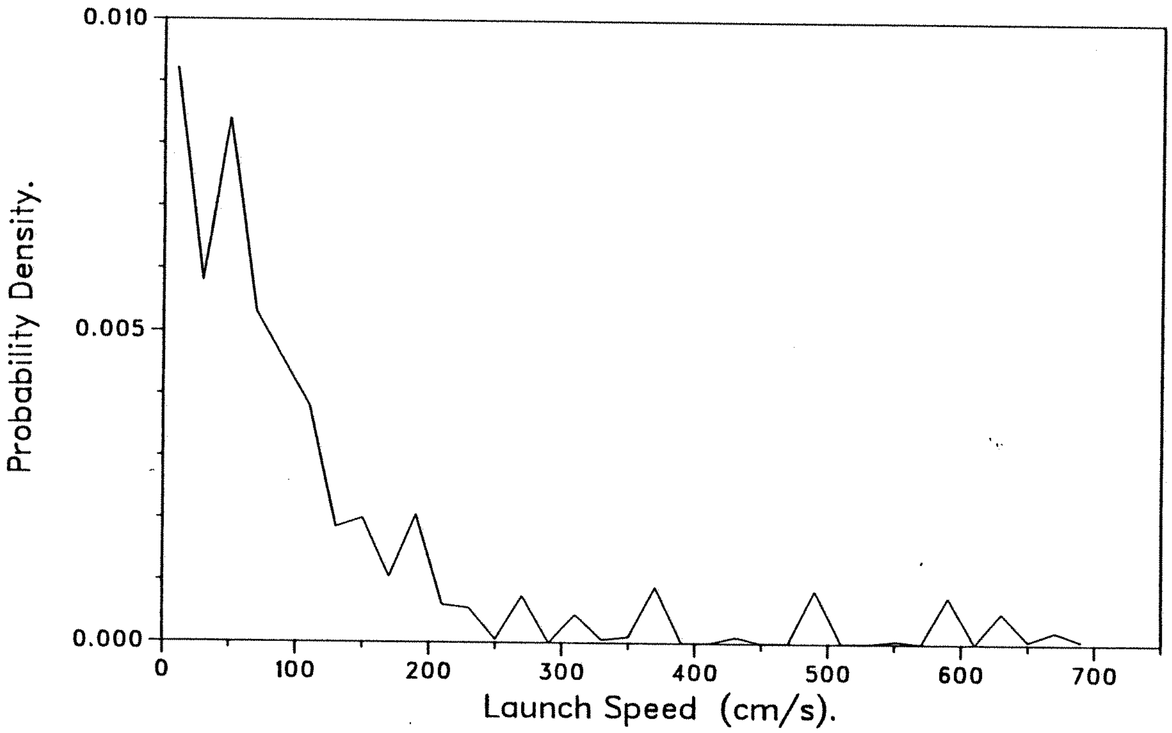


Fig.4.1. The estimated probability density function of the launch speed for the size class 211 - 258 μm in the Aarhus wind tunnel experiment.

approximated by an exponential distribution. In order to investigate this further a fractile diagram is made. If v_I denotes the length of the launch velocity vector and $\hat{F}(v_I)$ denotes the cummulative distribution function corresponding to the estimated density, we plot the points $(v_I, -\ln(1-\hat{F}(v_I)))$ for suitable values of v_I. In case the distribution of v_I is exponential these points should follow a straight line through the origin. The fractile diagram, which is given in Fig.4.2, is more smooth than the curve in Fig.4.1, as one would expect, and it shows that an exponential distribution

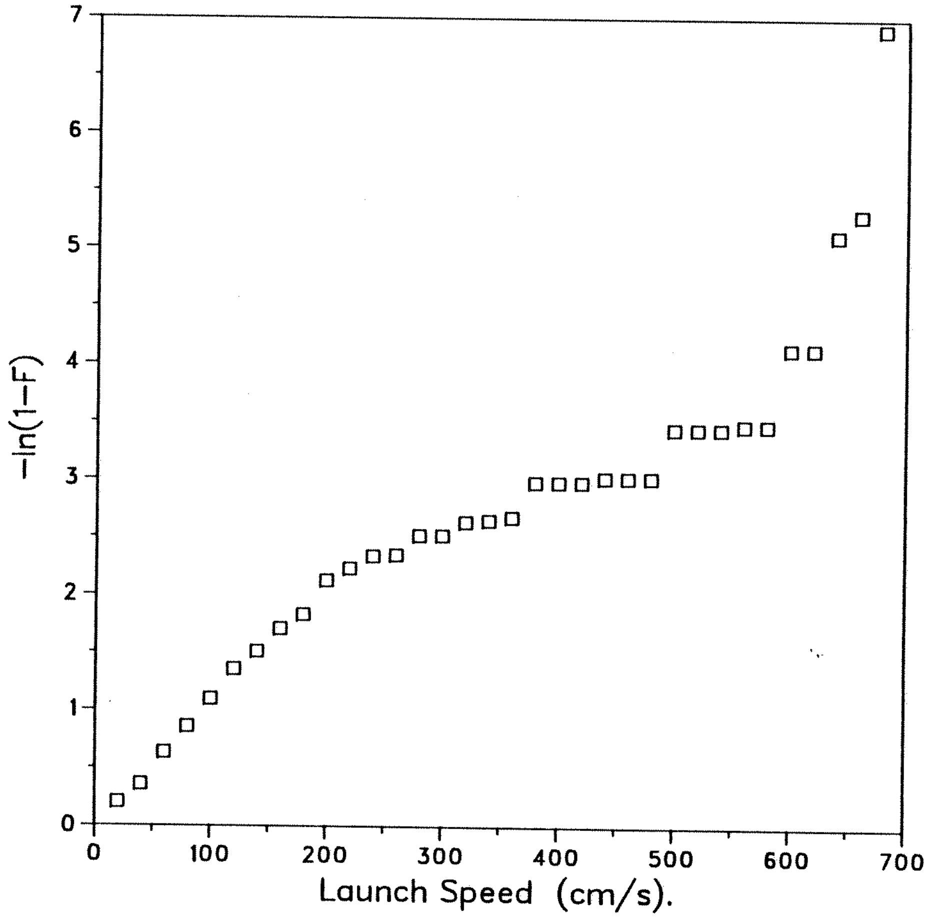
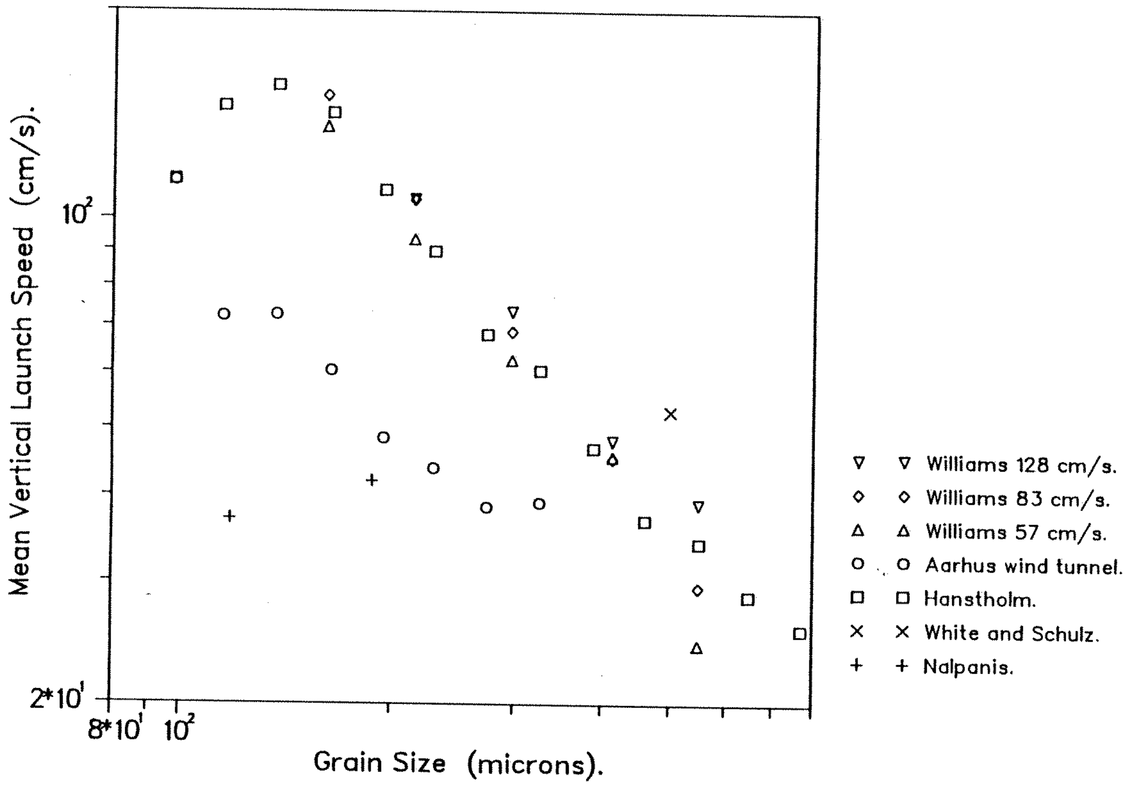


Fig.4.2. Fractile diagram comparing the estimated density shown in Fig.4.1 to the exponential distribution.

provides a good description of the estimated probability distribution, except for large launch speeds. The same general pattern is found for the other size classes too.

The mean value of the <u>vertical component of the launch velocity</u> for each individual grain size is plotted against the grain size in Fig.4.3. All five experiments are used. We have plotted the results by White and Schulz (1977) and Nalpanis (1985)



<u>Fig.4.3</u>. The vertical component of the estimated mean launch velocity for each individual size class. All experiments are plotted. Also direct observations by White and Schulz (1977) and Nalpanis (1985) are given in the diagram.

in the same figure. Our results are in good agreement with the results obtained by direct observation. In Nalpanis' experiments the friction velocities were in the range 18 - 21 cm/s, while White and Schulz's experiment was carried out at a friction velocity of 40 cm/s.

The vertical component of the launch velocity is clearly decreasing with the grain size in accordance with the results in Jensen and Sørensen (1985). Nalpanis' results are not in disagreement with this conclusion since his two points correspond to two narrow individual sand populations whereas our result concerns grains of different size within the same sand population. It is well known, see Zingg (1953), that the average jump height increases with the mean grain size of the sand population. This must also be true of the mean vertical launch speed.

The finding by Jensen and Sørensen (1985) that the mean vertical launch speed is approximately inversely proportional to the grain diameter is supported by Fig.4.3. Over a broad range of grain sizes the points in Fig.4.3 are well described by straight lines with slope -1. (Hanstholm: -1.1, Aarhus: -0.9, Williams: -1.3, -1.3 and -1.1.) The diagonal from the upper left corner to the lower right corner of Fig.4.3 has slope -1. Also the mean length of the launch velocity vector is found to decrease with grain size.

There is clearly a difference between the Aarhus results and the results from the other experiments. One possible explanation of this difference is that the mean vertical launch speed increases with the friction velocity until a certain level and then only depends slightly on the friction velocity. Results by

Gerety and Slingerland (1983) give some support to such an explanation and suggest that the hypothesized level might lie somewhere between the Aarhus experiment and the other experiments. Another possible explanation is the difference in mean grain size of the bed material. The effect of this was discussed above. Note, that Nalpanis' results are in accordance with both explanations. White and Schulz used glass spheres in their experiment, so their result can presumably not be used in this discussion. If it could be used it would support the second explanation, because the friction velocity was 40 cm/s only.

The estimated probability density functions of the launch angle for four size classes are plotted in Fig.4.4. Presumably

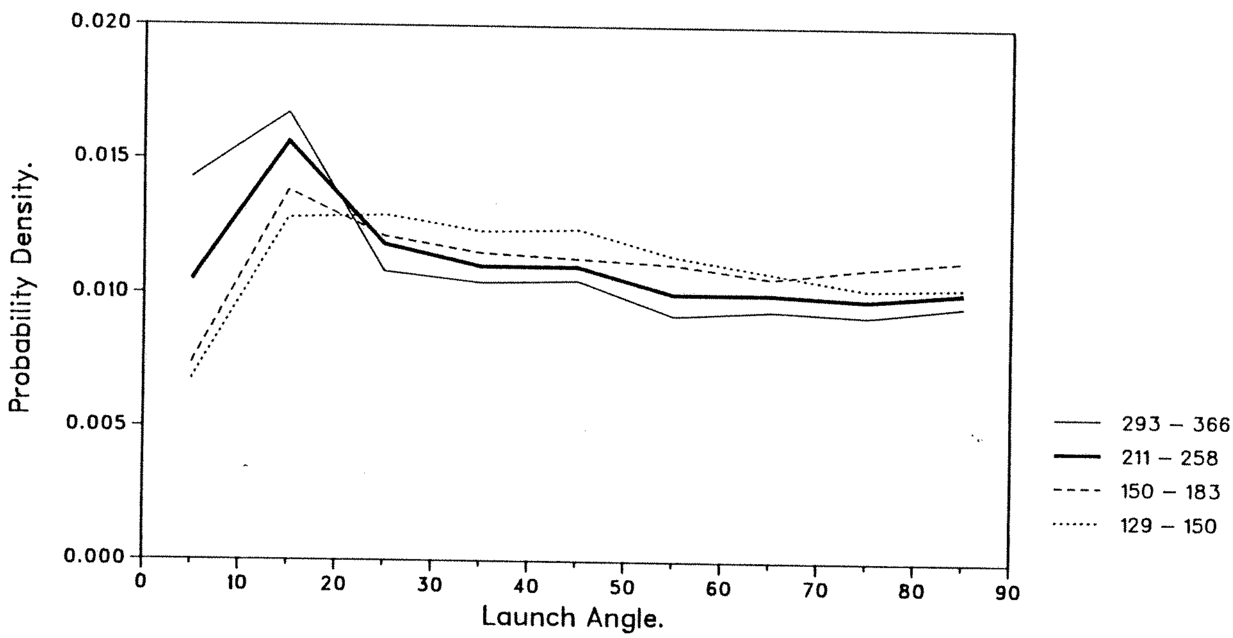


Fig.4.4. The estimated probability density function of the launch angle obtained from the Aarhus wind tunnel experiment for the four size classes indicated.

the shape of these densities is rather dependent on the estimation procedure used. However, they do show some trends and it is of interest to compare them to results obtained by direct observation. The densities have a peak at the small angle end of the distributions in accordance with the launch angle distributions found by White and Schulz (1977), Nalpanis (1985) and Willetts and Rice (1985). However, our densities decrease much less than those obtained by the other authors. Note, that the probability mass tends towards smaller angles as the grain size increases. This tendency is clearly seen in Fig.4.5, where the mean launch angles are plotted against grain size.

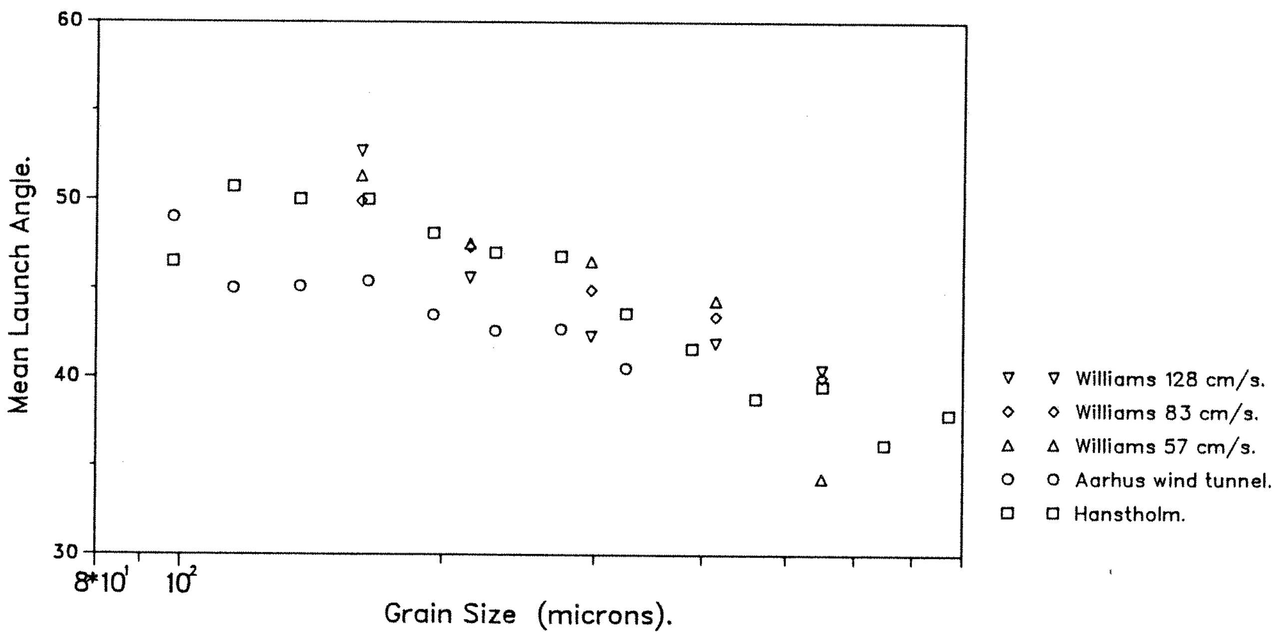


Fig.4.5. The estimated mean launch angle for each individual size class. All experiments are plotted.

Our estimates of the mean launch angle for the entire sand populations are in the range $43^{\circ}-46^{\circ}$. This should be compared to White and Schulz's (1977) value of 50°, Nalpanis' (1985) range of $34^{\circ}-41^{\circ}$ and the range $21^{\circ}-33^{\circ}$ of mean recochet angles found by Willetts and Rice (1985). The last authors found that grains dislodged from rest by impinging grains start at angles of $52^{\circ}-54^{\circ}$.

Finally, we shall briefly consider the relationship between the launch speed and the launch angle. In Fig.4.6 the conditional expectation of the vertical component of the launch velocity given the launch angle is plotted against the launch angle. For small

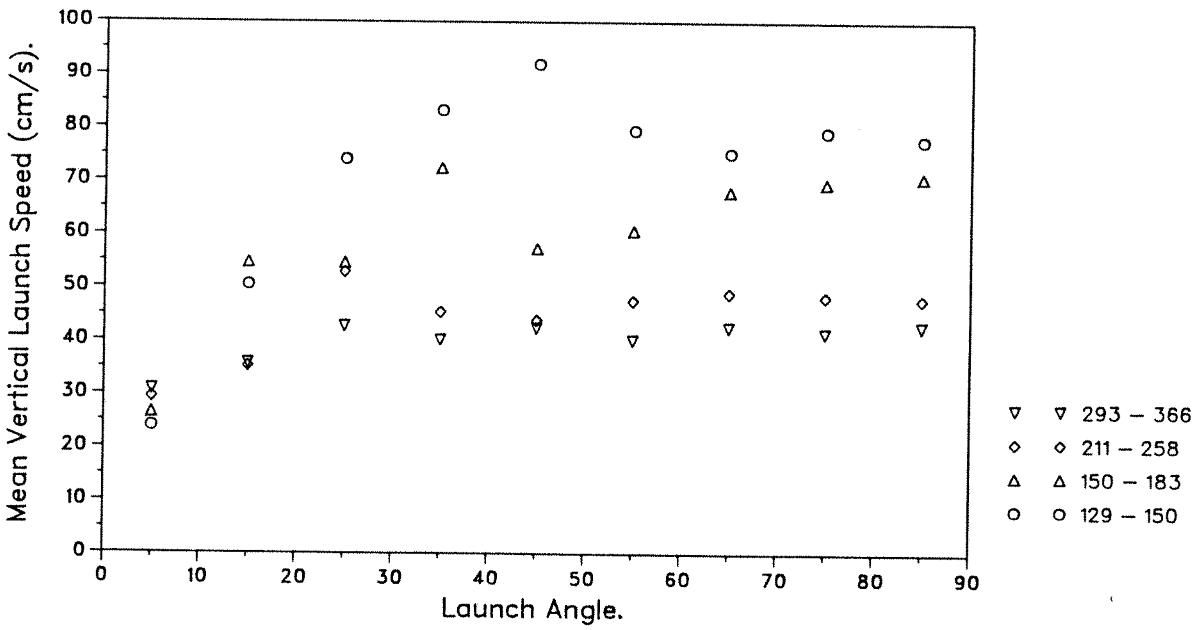


Fig.4.6. The estimated conditional expectation of the vertical component of the launch velocity given the launch angle. Only results from the Aarhus wind tunnel experiment are plotted.

angles the conditional expectation increases, but it is almost constant over a large range of angles. It is possible to describe the observed behaviour by curves of the type $a(1-\exp(-b\theta))$, where θ denotes the launch angle, while a and b are positive constants to be estimated. We will not pursue this problem further in this paper. Note, that the relationship shown in Fig.4.6 depends on the grain size.

5. The grain trajectories

We shall now consider some average characteristics of the grain trajectories. The estimated mean jump lengths for the entire sand populations are plotted in Fig.5.1 against the friction velocity. Nalpanis' (1985) results on the mean jump length is plotted in the same figure. The accordance between the results obtained by the two methods is good. We remind the reader that it was shown in Section 3 that the transport rate equals the product of the mean jump length and the dislodgement rate. Of course, mean jump lengths for the individual size classes can also be calculated

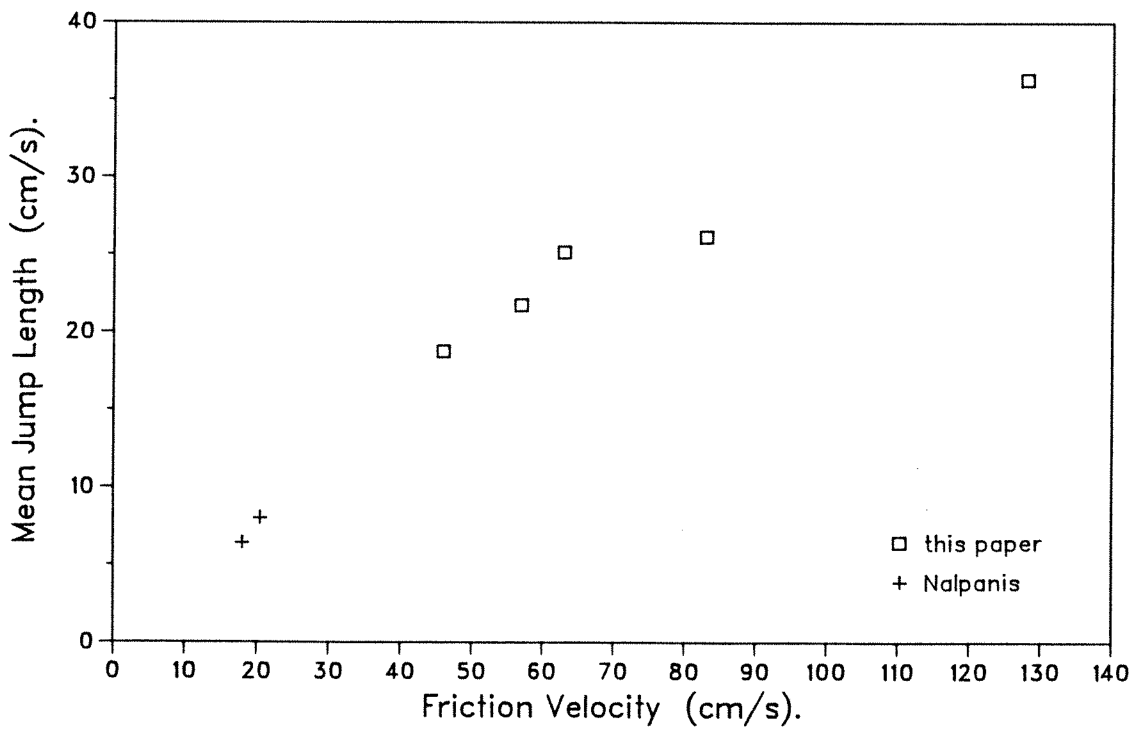
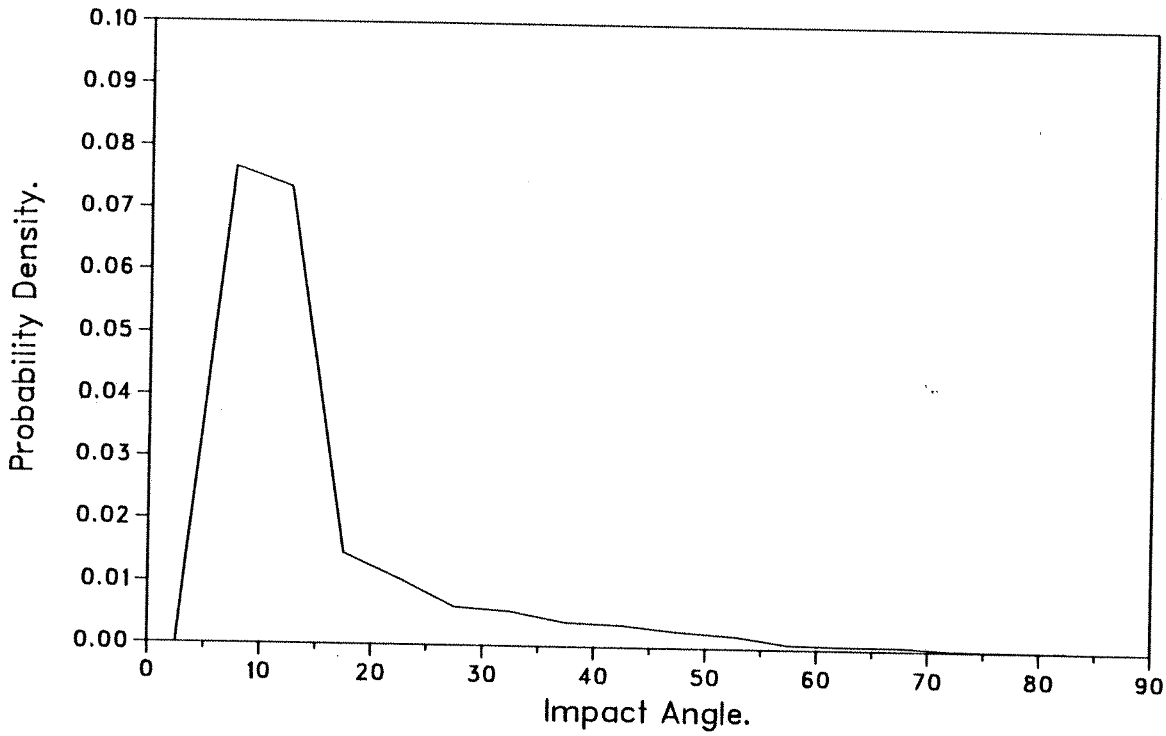


Fig.5.1. The estimated mean jump lengths for the entire sand populations plotted against the friction velocity for all experiments. Also Nalpanis' (1985) direct observations are plotted.

so that the difference in transport rate between the size classes can be studied. This will not be done here.

The probability density function of the <u>impact angle</u> for the entire sand population estimated from the Aarhus wind tunnel data is plotted in Fig.5.2. This density function resembles very much the densities reported by White and Schulz (1977) and Nalpanis (1985), except that it has a much heavier upper tail. Our mean impact angles are in the range $15^{\circ} - 25^{\circ}$ depending on the grain size. This can be compared to the mean value of 14° found by White and Schulz (1977) and the range $11^{\circ} - 14^{\circ}$ reported by Nalpanis



<u>Fig.5.2</u>. The estimated probability density function of the impact angle for the entire sand population in the Aarhus wind tunnel experiment.

(1985). A possible explanation of the fact that our estimates are larger than those obtained by direct observation is that some very low trajectories cannot be observed directly due to the dense saltation cloud. Very low trajectories have rather large impact angles, so this can perhaps also explain the heavy upper tail in Fig.5.2. It should, however, be kept in mind that the quality of the observed transport profiles used for our calculations is disputable close to the bed.

The mean impact angles for each individual size class, as estimated from Williams' data are given in Table 5.1. The mean impact angle increases with grain size and decreases with wind speed. A discussion of this finding can be found in Jensen and Sørensen (1985).

	U_* (cm/s)	57	83	128
grain size (µm)	161	13.2	11.3	9.0
	215	17.4	14.5	13.0
	297	22.7	20.1	16.2
	414	27.1	25.5	21.7
	550	31.0	31.1	26.1

Table 5.1. Mean impact angles estimated from Williams' (1964) experiments.

The probability distribution of the horizontal component of the velocity of a saltating grain that passes a fixed point in a particular height can also be calculated. Using the Aarhus wind tunnel data this has been done for three heights: 1.4 cm, 4.1 cm and 8.6 cm. In Fig.5.3 these probability distributions are given together with the similar distributions for respectively rising grains and descending grains only. Fig.5.3 is in qualitative accordance with White's (1982) and Greeley, Williams and Marshall's (1983) findings obtained by direct observation.

Greeley et al. (1983) found that the mean grain speed at a particular height did not depend much on the wind speed over a wide range of wind speeds. Contrary to this we find that the mean grain speed does depend on the wind speed, see Fig.5.4. The dependence of the grain speed on the wind speed complicates quantitative comparison between the results of this paper to those reported by White (1982) and Greeley et al. (1983). Differences in grain size is another complication. The only case where we find that a quantitative comparison is possible is between a field experiment carried out by Greeley et al. (1983) at Waddell Beach, California and the field experiment at Hanstholm. The typical grain size was the same at the two sites, whereas the wind speed was higher at Hanstholm (8.9 m/s at a height of 1 m) than at Waddell (6 m/s at the same height). A plot of the mean horizontal grain speed as a function of height, as obtained by our method from the Hanstholm data, is plotted in Fig.5.5 together with the direct observations from Waddell. Taking into account the difference in wind speed (cf. Fig.5.4) the accordance between the results obtained by the different methods is surprisingly good.

It would be very interesting to use simultaneously observed grain speeds at a number of heights and observed transport rate

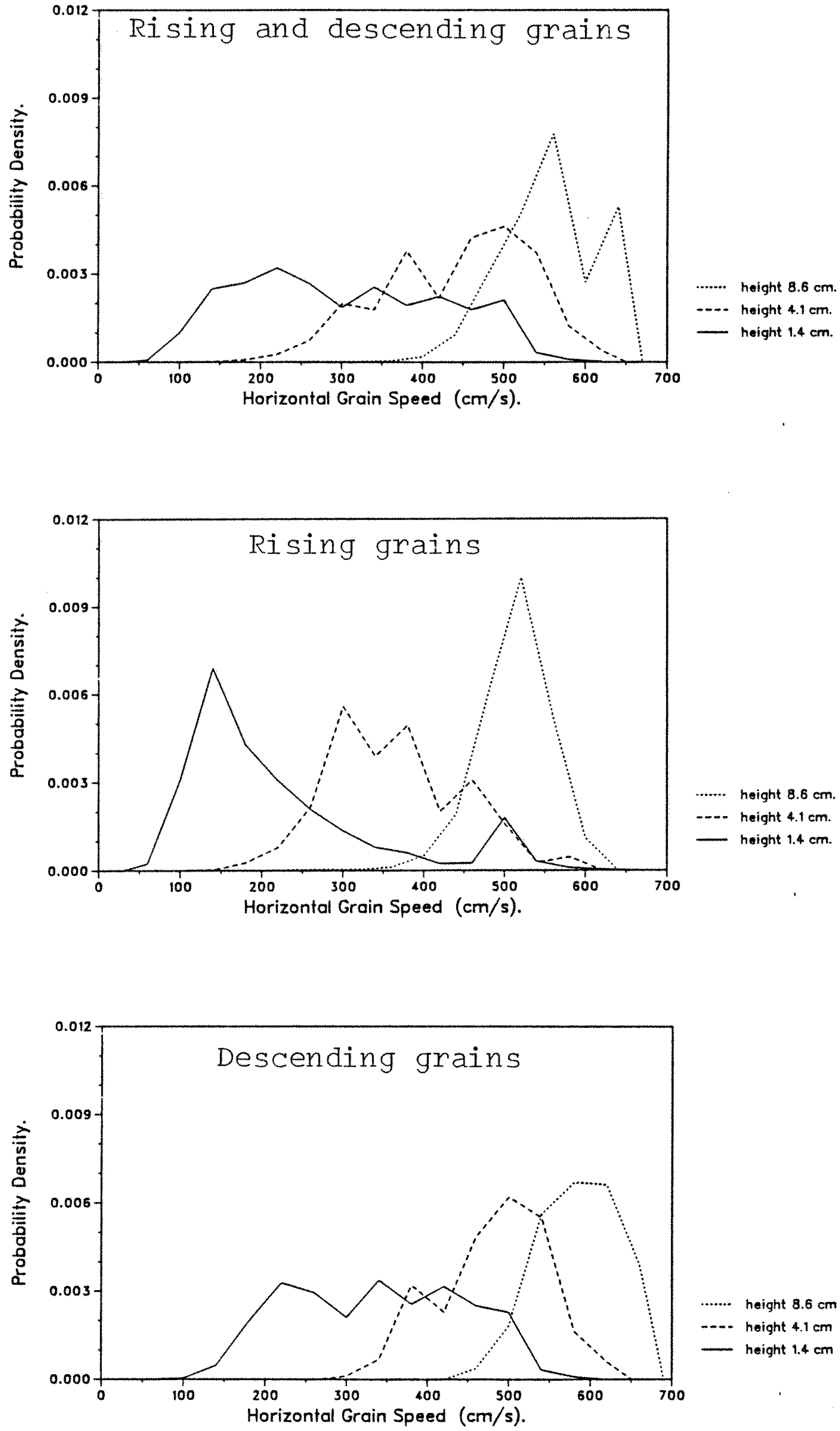


Fig.5.3. The estimated probability density functions for the entire sand population of the horizontal component of the grain velocity during saltation at the heights indicated. Also the density functions for rising grains and descending grains separately are plotted. Only results from the Aarhus wind tunnel experiment are plotted.

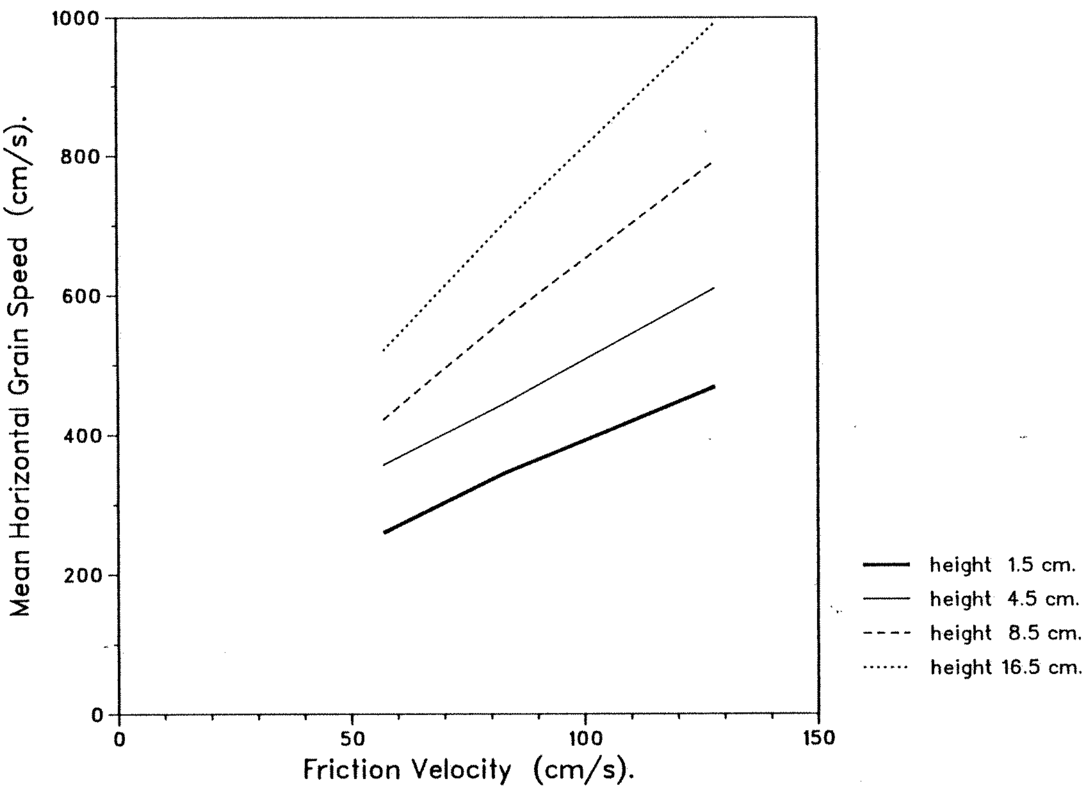


Fig.5.4. Estimated mean horizontal grain speed from Williams' experiments plotted against the friction velocity for the height indicated.

profiles to estimate the launch velocity distribution. These two kinds of data would clearly supplement each other very well.

In this section and in Section 4 we have compared our results to direct observations as far as this is possible. The accordance is good enough that we in the next two sections can embark upon calculating quantities that cannot easily be observed directly.

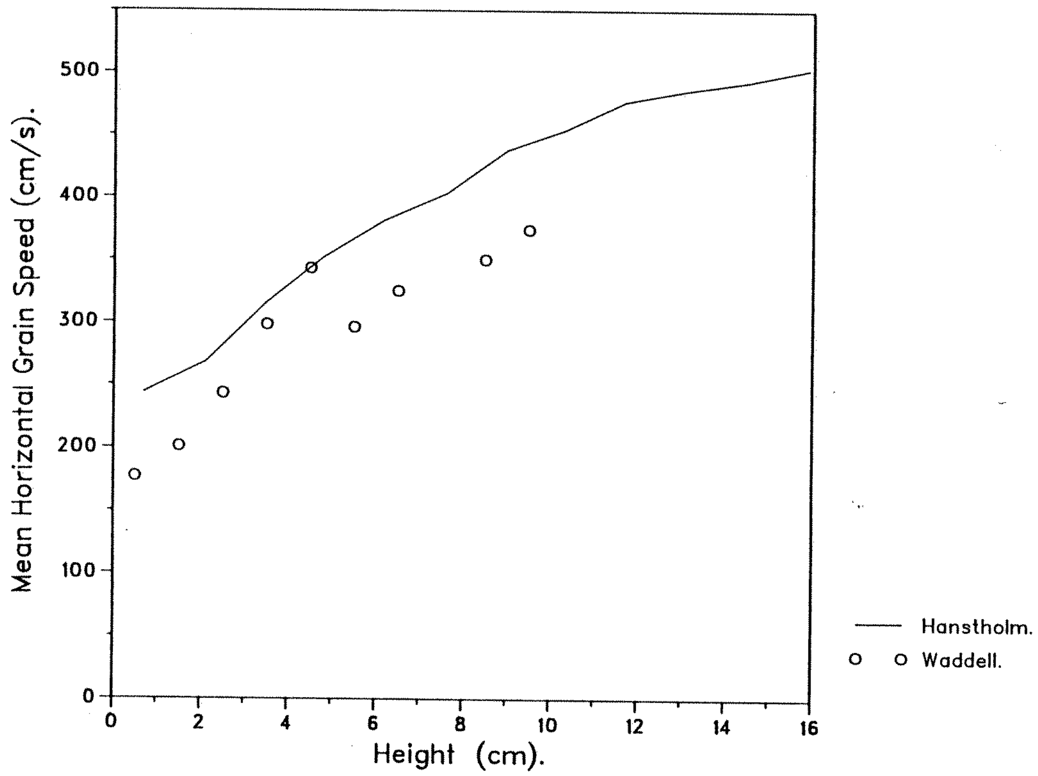


Fig.5.5. The estimated mean horizontal grain speeds for the Hanstholm experiment plotted as a function of height. Also the similar direct observations obtained at Waddell Beach by Greeley et al. (1983) are plotted.

6. The dislodgement rate

The total dislodgement rate, i.e. the mass flux of sand from the bed into the air, is plotted against the friction velocity in Fig.6.1 using all the experiments considered in this paper. A straight line gives a fair description of the variation of the points. This means that, to a fair approximation, the total dislodgement

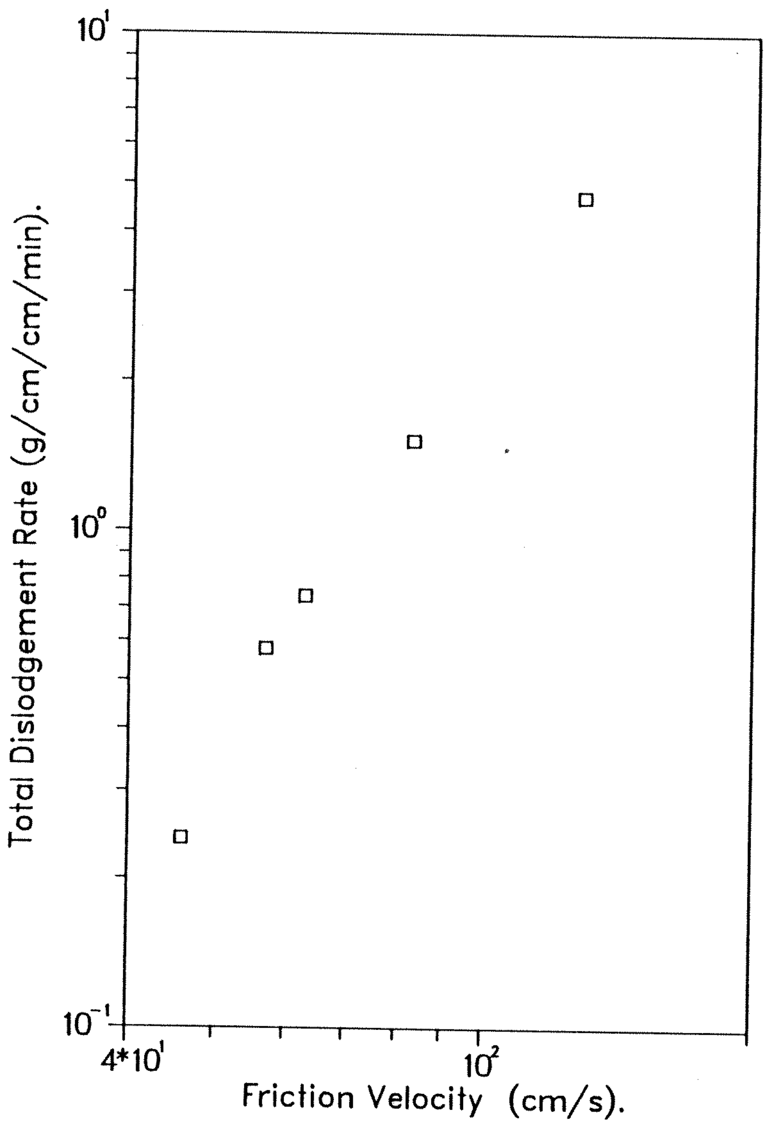
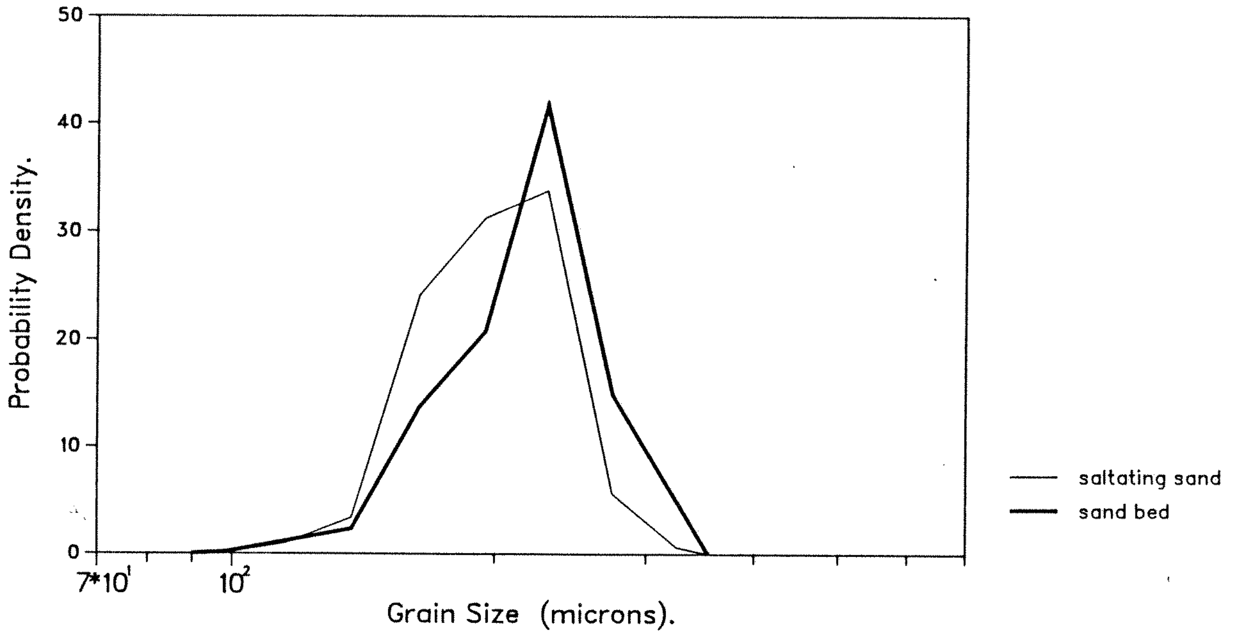


Fig.6.1. The total dislodgement rate plotted against the friction velocity for all experiments.

rate is proportional to $U_*^{2.8}$. This is in accordance with what was found in Jensen and Sørensen (1985).

In order to understand how the transport rate depends on the size distribution of the bed material and to explain changes in the sand sorting, it is important to consider the size distribution of the saltating sand, i.e. the dislodgement rate for each individual size class normalized by the total dislodgement rate. In Fig.6.2 the size distribution of the saltating sand is plotted together with that of the bed material for the Aarhus experiment and for Williams' data. A first thing to notice is that the size distribution of the saltating sand depends much more on the size distribution of the bed material than on the wind speed. At a friction velocity of 46 cm/s (Aarhus) the saltating sand is slightly finer than the bed material, whereas the saltating sand is coarser than the bed material at the larger friction velocities used by Williams. Thus the size distribution of the saltating sand seems to shift towards coarser grains as the friction velocity increases. Note however, that there is only a minor change from a friction velocity of 83 cm/s to one of 128 cm/s. At these friction velocities the saltation seems to have reached a stage where the size distribution of the saltating grains is almost independent of the wind speed.

Aarhus.



Williams.

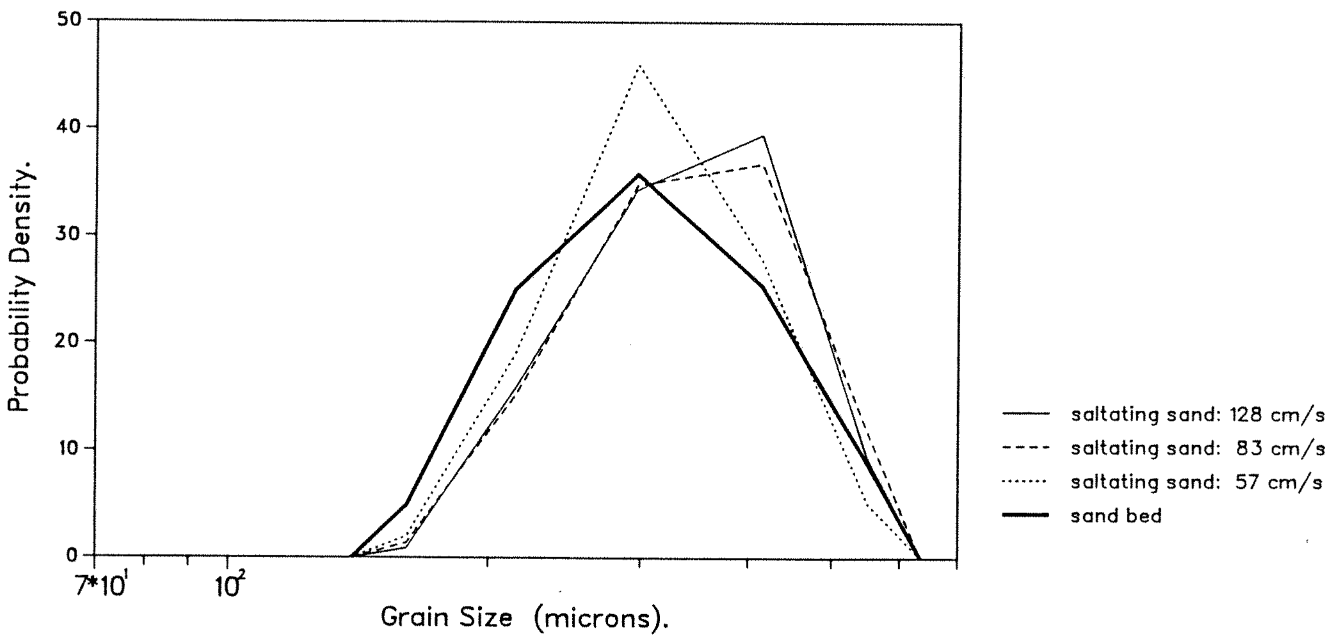


Fig.6.2. The size distribution of the saltating sand for the Aarhus wind tunnel experiment and for Williams' experiments.

7. On the interaction between wind and saltating grains

The concentration by mass of saltating grains at height y is given by

$$\int_0^\infty \int_{V_y} (v_1(y,\vec{v}_I,s)^{-1} - v_2(y,\vec{v}_I,s)^{-1})p(\vec{v}_I|s)d\vec{v}_I\phi(s)ds, \qquad (7.1)$$

with the notation of Section 3. This follows from (3.1) and (3.2). The concentration of saltating grains calculated from the Aarhus wind tunnel data by means of this formula is plotted against height in Fig.7.1. The concentration increases drastically close to the

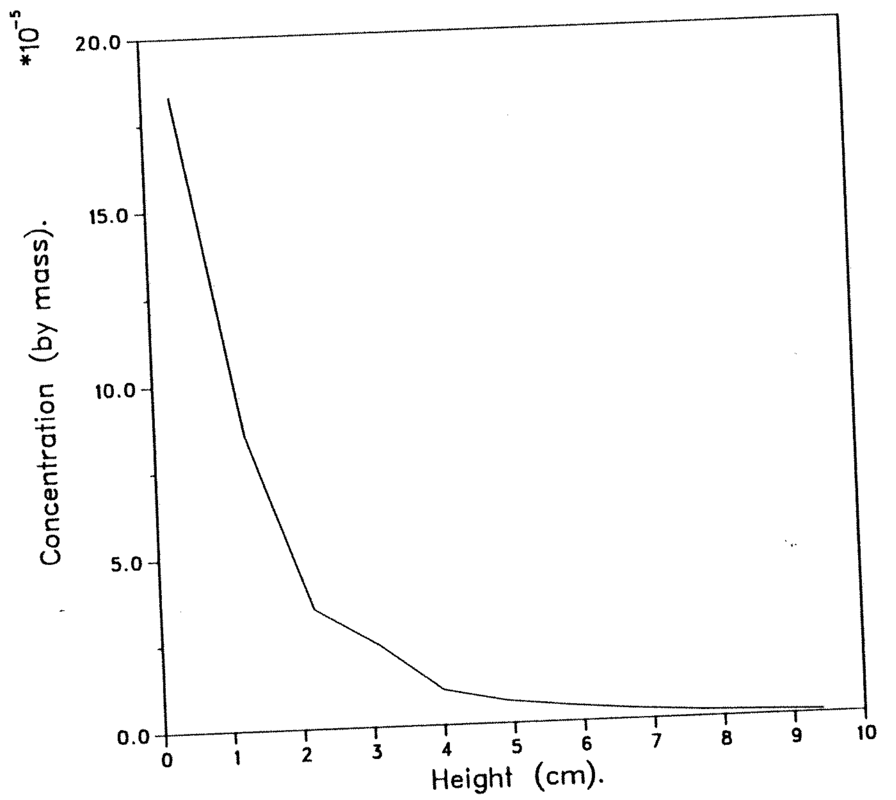


Fig.7.1. The concentration by mass (g/cm^3) of saltating grains plotted as a function of height for the Aarhus wind tunnel experiment.

bed, but even here it is very low: of the order 10^{-4} g/cm^3. Even for the largest friction velocity considered by Williams, i.e. 128 cm/s, the concentration close to the bed is of the order 10^{-3} g/cm^3 only. This implies that there is no reason to suspect that the grain concentration affects the drag coefficient, cf. Fig.9 in Bagnold (1956). Bagnold states that in the presence of saltating grains the drag coefficient is, to a round approximation, enlarged by a factor $(1-C)^{-3}$, where C is the volume concentration. The volume concentration is obtained by dividing the concentration by mass by 2.65, so even for a very high friction velocity we find that $(1-C)^{-3} = 1.004$.

We will now discuss how this grain concentration interacts with the wind. Owen (1964) introduced the notion grain borne shear stress. The grain borne shear stress at height y can be defined as the force exerted by the saltating grains on the air in a column with a base area of 1 cm^2 extending from the height y and upwards. The momentum extracted from the wind by the grains above the level y is transferred by the grains down below the level y. The grain borne shear stress at height y_0 is given by

$$T(y) = \int_0^\infty \int_{y_0}^\infty \int_{V_y} \Big[\frac{D(w_1(y,\vec{v}_I,s))(U(y)-u_1(y,\vec{v}_I,s))}{v_1(y,\vec{v}_I,s)w_1(y,\vec{v}_I,s)} - \frac{D(w_2(y,\vec{v}_I,s))(U(y)-u_2(y,\vec{v}_I,s))}{v_2(y,\vec{v}_I,s)w_2(y,\vec{v}_I,s)}\Big]\frac{\phi(s)p(\vec{v}_I|s)}{m(s)}d\vec{v}_I\,dy\,ds, \quad (7.2)$$

where $w_1(y,\vec{v}_I,s)$ and $w_2(y,\vec{v}_I,s)$ are the length of the vector of relative velocity between grain and air at height y for a grain of size s that started with launch velocity $\vec{v}_I$ while it rises and descends respectively. The quantity $m(s)$ is the mass of a grain of size s, and the rest of the notation was defined in Sections 2 and 3. Now, define $t_1(y_0,\vec{v}_I,s)$ and $t_2(y_0,\vec{v}_I,s)$ as the two moments where a grain of size s passes the level y provided its launch velocity $\vec{v}_I$ allows these events. Then we can simplify (7.2) by means of equation (2.1):

$$T(y)$$

$$= \int_0^\infty \int_{V_{y_0}} \int_{t_1(y_0,\vec{v}_I,s)}^{t_2(y_0,\vec{v}_I,s)} \ddot{x}(t,\vec{v}_I,s)dtp(\vec{v}_I|s)d\vec{v}_I\phi(s)ds \qquad (7.3)$$

$$= \int_0^\infty \int_0^\infty \Delta\dot{x}(y_0,\vec{v}_I,s)p(\vec{v}_I|s)d\vec{v}_I\phi(s)ds$$

$$= \phi_T \overline{\overline{\Delta\dot{x}(y_0)}},$$

where ϕ_T is the total dislodgement rate, and $\Delta\dot{x}(y_0,\vec{v}_I,s)$ is the increase in horizontal speed for a grain of size s and launch speed $\vec{v}_I$ between the moments $t_1(y_0,\vec{v}_I,s)$ and $t_2(y_0,\vec{v}_I,s)$. The quantity $\overline{\overline{\Delta\dot{x}(y_0)}}$ is the average increase in horizontal speed for a grain while it is above the level y_0. Note, that in particular this shows that the grain borne shear stress at the sand surface equals the product of the total dislodgement rate and the average amount of momentum extracted from the wind by a saltating grain.

The grain borne shear stress at the surface is given in Table 7.1 together with the shear stress, ρU_*^2, in the grain-free logarithmic layer above the saltation layer. Our estimates of the grain borne shear stress at the surface are considerably smaller than the value supposed by Bagnold (1941) and Owen (1964) in their theories of saltation, see the discussion in Jensen and Sørensen (1985). The variation of the grain borne shear stress with height is plotted in Fig.7.2 for the Aarhus wind tunnel data. Close to the bed the grain borne shear stress amounts to only 14 per cent of ρU_*^2, and at a height of 4 cm only 2 per cent.

	grain borne shear stress	ρU_*^2
Aarhus	0.4	2.6
Hanstholm	0.5	4.9
Williams	0.6	4.0
	1.8	8.5
	8.1	19.9

Table 7.1. Grain borne shear stress ($g\,cm^{-1}\,s^{-2}$) calculated by (7.3) and the shear stress, ρU_*^2 ($g\,cm^{-1}\,s^{-2}$), in the grain-free logarithmic layer.

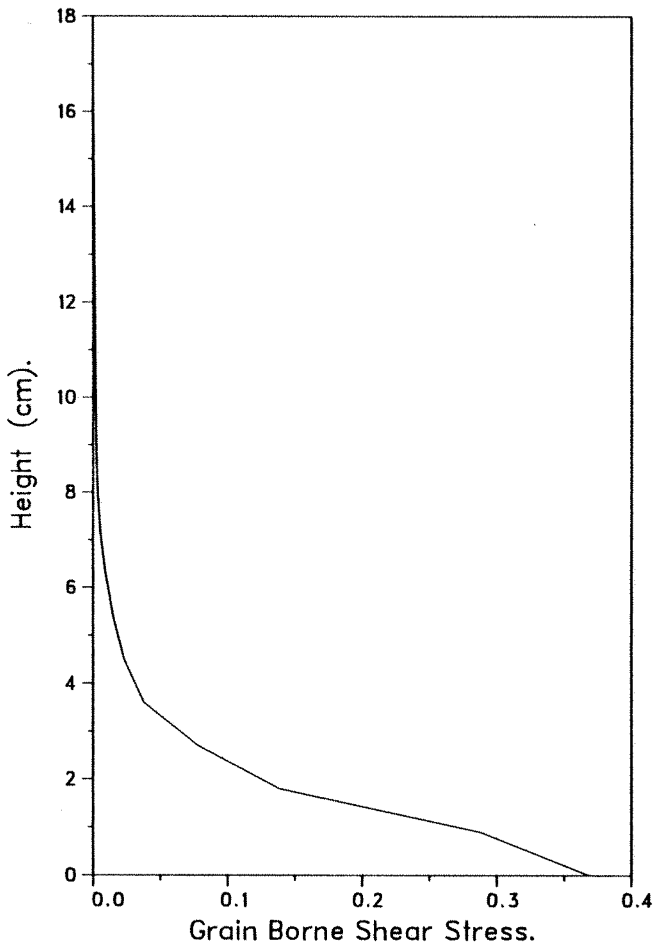


Fig.7.2. The grain borne shear stress plotted as a function of height for the Aarhus wind tunnel experiment.

In order to calculate how the grains affect the wind speed we have to make two further assumptions. The first is that the sum of the grain borne and the air borne shear stresses, i.e. the total shear stress, is constantly equal to ρU_*^2 in the saltation layer. This assumption is also made when the logarithmic wind speed profile over a fixed surface is derived. It implies that the air borne shear stress $\tau(y)$ at height y is given by

$$\tau(y) = \rho U_*^2 - T(y). \tag{7.4}$$

The second assumption is that we can calculate the wind velocity profile by means of an eddy viscosity, $\nu(y)$, and that $\nu(y)$ equals the eddy viscosity in a grain-free logarithmic layer, i.e.

$$\nu(y) = \varkappa U_* y, \tag{7.5}$$

where $\varkappa$ is von Karmans constant. Using (7.4) and (7.5) we find that

$$\rho U_*^2 - T(y) = \rho \varkappa U_* y \frac{dU}{dy},$$

and hence by (7.3)

$$\frac{dU}{dy} = \frac{U_*}{\varkappa y} - \frac{\phi_T \overline{\overline{\Delta \dot{x}(y)}}}{\rho \varkappa U_* y} .$$

By integration we obtain the mean velocity profile

$$U(y) = 2.5 U_* \ln(y/y_0) + \frac{\phi_T}{\rho \varkappa U_*} \int_y^{y^*} \frac{\overline{\overline{\Delta \dot{x}(z)}}}{z} \, dz, \tag{7.6}$$

where y^* is a height where the concentration of sand grains is zero.

The first term in (7.6) is a continuation into the saltation layer of the logarithmic velocity profile found in the grain free layer above the saltation layer. The second term corrects for the effect of the grains. In Section 6 we found that $\Phi_T \propto U_*^{2.8}$, and in Jensen and Sørensen (1985) it was found that Φ_T does not depend much on the size distribution of the sand bed. However, $\overline{\overline{\Delta\dot{x}(y)}}$ varies with U_* as well as with the size distribution of the bed material in a complex way. In order to obtain an expression for the variation of the correction term in (7.6) a further study of the variation of $\overline{\overline{\Delta\dot{x}(y)}}$ is needed. Of course, equation (7.6) should also be tested against data.

In Fig.7.3 the wind velocity profile calculated by (7.6) from the Aarhus wind tunnel data is plotted together with the logarithmic velocity profile. We see that only very close to the surface is the correction term of importance. At higher wind speeds the difference becomes more pronounced. A similar plot

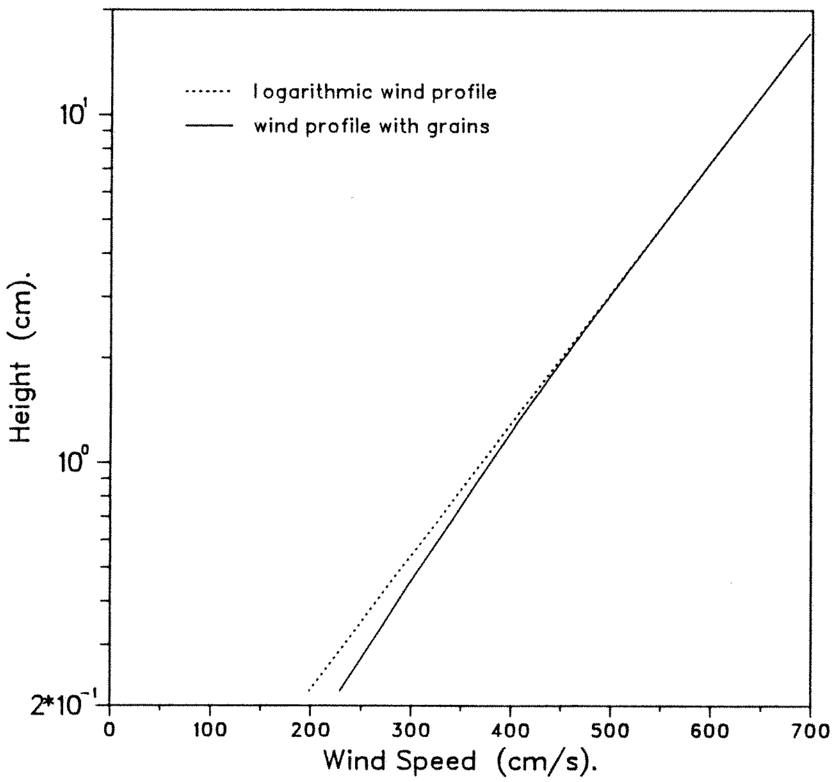


Fig.7.3. The wind velocity profile in the saltation layer as calculated by (7.6) from the Aarhus wind tunnel experiment. Also the logarithmic wind velocity profile is plotted.

for the Hanstholm data is given in Rasmussen, Sørensen and Willetts (1985). In the same paper the correction term as estimated from all experiments considered in this paper is plotted as a function of the friction velocity for three different heights. A careful study of the available data using a mathematical model as the one proposed above could perhaps cast some light on the problem of how to determine U_* from wind speed measurements, see the comprehensive discussion in Gerety (1985).

Owen (1964) proposed to use a constant eddy viscosity in the saltation layer. This would result in a considerably larger modification of the logarithmic wind profile than implied by (7.6).

8. Conclusions

Observed aeolian sand transport rate profiles were analysed by means of a stochastic saltation model. The most important findings were:

1) Estimates were obtained for the probability distribution of the launch velocity vector, the mean jump length, the mean impact angle and the probability distribution of grain speed during saltation at a fixed height. These estimates were in fair accordance with results obtained by direct observation.

2) In accordance with results in Jensen and Sørensen (1985), the vertical component of the mean launch velocity decreases with grain size approximately inversely proportional to the grain diameter.

3) The size distribution of the saltating sand depends much more on the size distribution of the bed material than on the wind speed. The saltating sand becomes coarser as the wind speed increases.

4) As in Jensen and Sørensen (1985) the estimated grain borne shear stress at the surface is smaller than assumed by Bagnold (1936) and Owen (1964).

5) A method for calculating the wind velocity profile in the saltation layer was proposed.

Acknowledgement

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Michael Sørensen
Department of Theoretical Statistics
Institute of Mathematics
University of Aarhus
Building 530
Ny Munkegade
DK-8000 Aarhus C
Denmark