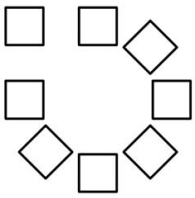

Centre for Symmetry and Deformation

Department of Mathematical Sciences, University of Copenhagen

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Final Report 2010-2019

It is not without pride that I look back at the accomplishments of the Centre for Symmetry and Deformation, since its inception in 2010. In the span of 10 years, we have firmly established ourselves as a dominant European research entity within our field; we can celebrate a number of groundbreaking research results, and have through sustained recruitment efforts placed Copenhagen centrally on the map as a top postdoc and PhD researcher training site within Europe. In the process we are setting the stage for a new generation of stellar young researchers to capitalize on the legacy of the center, to carry out their own research agendas.

RESEARCH: The center has produced an outstanding body of research: We managed to consistently make breakthroughs worthy of the extremely selective top 5 mathematics journals, with a total of 12 papers in those journals so far, and expectations of more to come. The specific distribution so far is: 2 in *Ann. Math.*, 2 in *J. Amer. Math. Soc.*, 4 in *Invent. Math.*, 3 in *Acta Math.*, and one in *Publ. Math. IHES*, and a total of 463 papers on the arXiv under our tag CPH-SYM-DNRF92. (The time from conception to preprint, as well as preprint to acceptance, in mathematics can take years.) We have also received many of the standard markers of the highest scientific excellence during the center duration: 2 talks at the quadrennial *International Congress of Mathematics*, 4 ERC grants, and a host of other international prizes and prestigious lecture series, as well as Danish prizes such as the *Elite Researcher Prize*.

The center has resoundingly demonstrated that deformation, or homotopy, methods can be used to effectively describe a range of phenomena about symmetries, be it symmetries of geometric objects, or symmetries and their representations more abstractly. In each case it turns out that it is an important insight that one should not only remember the end result, but also keep track of how one got there, i.e., one should remember the homotopies. This "higher" approach to symmetries turns out to be applicable to a range of mathematical disciplines, spanning algebra, geometry, and analysis, and beyond, and the center has played a central role in capitalizing on that insight.

We here highlight the following 5 breakthrough results:

- Recall that an n–dimensional manifold is a geometric object that around each point looks like ordinary n–dimensional Euclidian space; e.g., the surface of a ball is an example of a 2–dimensional manifold. The work of Galatius and Randal-Williams, published as a series of three papers spanning 249 pages in *Acta Math.*, *J. Amer. Math. Soc.*, and *Ann. Math.*, and the related work of Berglund–Madsen, gives a general asymptotic description of the space of symmetries of all such objects.
- Higman–Thompson groups are groups of symmetries of fractal-like objects, that occur throughout mathematics. In breakthrough work, accepted in *Invent. Math.*, Szymik and Wahl determined the homology of such groups, in particular resolving a conjecture from 1992 of K. Brown.

- The stable homotopy category consists of geometric objects and certain functions between them, and underlies much of modern topology. A celebrated result from the 1980s gives a maximal stratification of this category, the so-called chromatic stratification, dividing it up into more manageable pieces. A central question has been to extend this stratification to a setting where one also takes into account symmetries. In two papers in *Invent. Math.*, postdocs Barthel, Hausmann, and Sanders, and their coauthors, managed to largely solve this question, computing the spectrum of the equivariant stable homotopy category.
- Symmetries that can be represented as symmetries in vector spaces are of fundamental importance, but can in general be quite difficult to determine. A well-understood case is when the vector space is 1–dimensional, but even the case when it is "almost-1–dimensional", i.e., the so-called endotrivial modules, is not properly understood. Resolving this impasse, Grodal in *arXiv:1608.00499*, found a way to describe such modules via homotopy, or deformation, methods, in particular solving the Carlson–Thévenaz conjecture.
- C^*–algebras are to be thought of as non-commutative spaces, and have their origin in quantum mechanics, where position and velocity cannot be simultaneously determined. An important class of C^*–algebras are those constructed from graphs. In a series of long papers, culminating in *arXiv:1611.07120*, Eilers and coauthors completed the classification of unital graph C^*–algebras.

In terms of comparison group, the leading places in the US in our field are Harvard, MIT, Stanford and Chicago, and leading places in Europe, other than us, include Bonn, Oxford, Münster, and more recently Cambridge. We would rank ourselves in the top layer of this very select group. In terms of getting papers in top journals, the track record listed above is certainly hard to beat. We also have a young and dynamic group, and having had Hesselholt's adjacent Bohr professorship has certainly added to our strength. That our last permanent hire Galatius chose Copenhagen over Stanford speaks for itself.

ACTIVITIES: Looking back at the lists of activities over the last 10 years, we can see why it has seemed quite busy around here: Our conference page `sym.math.ku.dk/conferences/` lists **40** conferences/workshops and **42** masterclasses. Our guest database `sym.math.ku.dk/guests` lists **1456** individual guests since 2010, staying for variable durations, where a guest is someone who has received an office during their stay, i.e., not counting casual conference participants. Many of our conference series have now become institutions in themselves: the bi-annual Young Topologists and Young Mathematicians in C^*–algebras meetings in Copenhagen have hundreds of participants and are well-established brands—we hope funds will be found to continue these activities.

PHD AND POSTDOC PROGRAM: **63** postdocs and **52** PhD students have been employed at the center, most for 3 years. The majority of the center budget has gone to fund these positions. Since in mathematics the most precious capital is brain-power, we have from day one realized the importance of good hiring procedures, both in advertisement and selection. Here predictability is key: Our postdoc call has each year been published with application deadline around December 1 for start the following September, and our PhD call twice annually in December and May, with start likewise in September. We have each year made approximately 5

hires in each category, as a result of perhaps twice as many offers, always going after the best candidates irrespective of area. PhD and Postdocs have moved on to find permanent positions throughout the world, including Stockholm (×2), Gothenburg, Trondheim, Cambridge, Warwick, Aberdeen (×3), Utrecht, Leuven, Angers, Louisiana, Maine, Irvine, Santa Cruz, Shanghai, Chiba, ISI-Kolkata, along with non-permanent and industry positions. As our reputation has increased, we have gotten stronger applications, and in greater numbers, and it has become easier to win in competition with other top places in Europe and the US. In short: we have become an admired *brand*. The taxpayers should also be thrilled to know that a very large proportion of foreign researchers who leave academia choose to remain in Copenhagen in industry jobs—the benefits of the confluence of a strong university and an attractive city.

EDUCATION AND OUTREACH: Education has been an integral component of the center, and due to our broad scientific reach, the center has touched a substantial part of the courses in the math department. The presence of the center has resulted in a notable increase in the number of international masters students, and we have had e.g., some of the best French ENS students do their undergraduate internship in Copenhagen, aiding later PhD recruitment. We have worked methodically with the department over the years to build up and refine our course offerings. The result is complete and clear course-lines within algebra, topology, and non-commutative geometry (see e.g., `https://tinyurl.com/ucph-top`). Since these have been integrated in the department they will be a benefit for everyone for many years to come. We have conducted a steady stream of outreach activities, as detailed in our annual reports. To give an example, Eilers famously corrected the official 1974 LEGO count for the ways to combine six 2×4 bricks; and now when exiting the new LEGO house in Billund, you are explained the count and handed your own unique 6-brick combination, taken from Eilers' 915,103,765 ways of combining them.

FUTURE: It feels in many ways odd to write this obituary at the height of the center's mathematical influence, and just after the recruitment of an absolutely stellar team. It is naturally a perilous moment—running a top level mathematics unit costs money—and retention, also of professors, could be in jeopardy; strong hires will have many options internationally, and it is always attractive to be where the resources are.

If we look to our competitors, the schools in the US that we compete against, Harvard, MIT, Stanford, Chicago, etc. all have fully department funded PhD and postdoc programs—they have been running the equivalent of a mega-center, all internally funded, continuously the last 50 years! South of the border in Germany, our competitors in Bonn have had their Max Planck Institute since 1980, and they also just had their gigantic "Hausdorff Center" University Excellence Cluster from 2006 renewed for another 7(+7) years. Münster was just added to the list of Excellence Clusters starting this year.

Of course transitional periods can also lead to growth; it can be an opportunity to regroup, retune your goals, and emerge stronger. The younger professors at the center are of an incredible caliber, with a track record of attracting e.g., ERC grants, and possess a host of international accolades. They interact closely with researchers in neighboring fields, both in geometry and higher algebra, where the viewpoint of the center is making its inroads. In our estimate the center has nourished talent and research directions enough to contain the germs for not just one, but several internationally leading research endeavors of a center magnitude. I hope that they, like us, will be generously funded!

11 March, 2019

Jesper Grodal
Professor, Center Director