NumDiff: Numerical Solution of Differential Equations Block 3A, 2012

What's happening in NumDiff?

NumDiff is a course of the type that makes Copenhagen University [KU] what it is:
"Strong on theoretical foundation".
KU graduates not only are able to apply the newest methods – they also understand what lies behind so that they can participate in creating the methods of the future.

We do see some "real life" applications through examples and weekly assignments, but the course focus is on the theoretical content of the methods - **comparing and evaluating various methods for the same problems with respect to for example cost and precision**.

NumDiff contains the **classical numerical (Finite Difference) methods for the solution of differential equations in both one and more dimensions (ordinary and partial differential equations)** only briefly touching on the more advanced integral methods (Collocation and Finite Element).
You learn the notions and the methods that all else is built on top of.
These are methods in practical use today but more often they are the foundations of the state of the art methods being used and researched today.

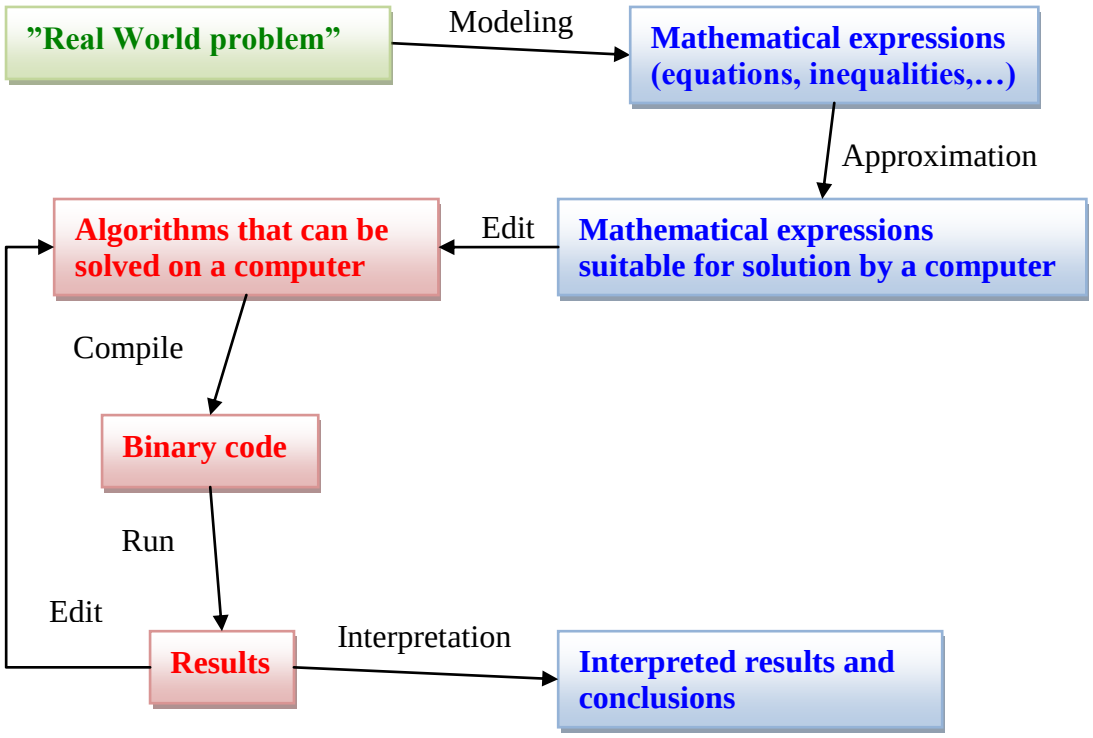
In NumDiff we also try out the methods - **writing and running code**.
We focus on "toy-problems" small enough that it is possible to focus on the methods and not get caught in "size-problems".
Follow up courses to NumDiff could be named "NumDiff - Size matters" or "NumDiff – Integral methods".

Why is NumDiff important?

You learn programming in practice.
You learn the basic foundation of the numerical solution of dynamic problems.
Applications:

1. **During your studies:** Within *insurance*, *finance*, *operations research*, *physics*, *chemistry*, *biology*, *and you name it* - you often need to solve your models on a computer. Whenever the problems are dynamic (changes with time or any other parameter) this will, one way or the other, involve the methods learned in this course.
2. **After your studies:**
    a. If you become a *researcher* you will need to solve your own dynamic models or help other solve theirs.
    b. If you become a *high school teacher* you will need the programming and numerical skills in interdisciplinary projects.
    c. If you go to *industry*, you will need a training period. In this period you will typically "earn your salary" as a programming help for a project group.

What is Numerical Analysis?



Focus in NumDiff:

1. **A little bit of "modeling".**
   The textbook contains examples taken from real life and the weekly assignments are originating in problems taken from real life.
2. **A lot of "Approximation"** in the cases where the mathematical expressions involve differential equations and large linear equation systems.
3. **Quite a few "Edit-Compile-Run-Error correct"-cycles** on expressions of the types above. **Maple** is used.
4. **Quite some "Interpretation" of results**, in the weekly assignments.

Lecturers:

- Jon Sporring (JS)
- Jens Hugger (JH, Contact person)

Teaching Assistants (TA's):

- Jens Petersen (JP, Assistant)
- Sarah Niebe (SN, Assistant)

Course contents:

The goal is that you learn some basic tools needed for solving mathematical problems on a computer:

1. Learn the necessary mathematics for solving differential equations in one or more dimensions on a computer using the "Finite Difference Method".
2. Extend your programming skills on how to make a computer run the mathematical algorithms that you learn to construct. The "Programming language" that we use is Maple. Also students with poor Maple skills are helped but must expect a steeper learning curve.

The topics are as follows:

Math-topics:

1. Background stuff from **mathematics**
2. Finite difference methods for **initial value problems for ordinary differential equations**
3. Finite difference methods for **boundary value problems for ordinary differential equations**
4. Finite difference methods for **diffusion problems for partial differential equations**
5. Finite difference methods for **advection problems for partial differential equations**
6. Finite difference methods for **wave problems for partial differential equations**
7. Finite difference methods for **Elliptic problems for partial differential equations**

Maple-topics:

1. **Basic Maple**
2. **Flow control**
3. **Procedures**
4. **Input/output**

Compulsory literature (Text book):

- **Mark H. Holmes**, "Introduction to Numerical Methods in Differential Equations", 2007, Springer, Texts in Applied Mathematics TAM-52, ISBN 0-387-30891-1. (Buy it)
  Note the accompanying webpage. (See folder "Course material").
- **Maple, Programming guide**, ch. 1-6, 9. (Free on KUnet->Softwarebiblioteket->Maple)
- **Lecture notes** supplementing but not replacing the book. You have to read the book and listen to the lectures. If you have problems with the book maybe the answers are in the lecture notes. (Free on Absalon. See folder "Course material"):
  - Jens Hugger, Maple theory lecture notes.
  - Jens Hugger, NumDiff lecture notes for lecture 4-5, 2011 (Holmes ch. 2) in Maple.
  - Jens Hugger, NumDiff lecture notes for lecture 7-14, 2011 (Holmes ch. 3-6) in pdf.
  - Maple Worksheets for lecture notes slide 10.3 and 10.5.

Supplementary reading:

For ch. 1-2:

- Jens Hugger, Numerical solution of DEP, 2007. (See folder "Course material").

On PDE's:

- PDE survey on Scholar Pedia
- J. W. Thomas, "Numerical Partial Differential Equations: Finite Difference Methods", Springer TAM 22.

On Maple:

- Maple, User guide. Maple, Programming guide. (Free on KUnet->Softwarebiblioteket->Maple).

Confrontation and home work:

Listening, theory training and reading:

Each week starts with an introduction Tuesday 8-9 rounding off the previous week and commencing the current week. **Attend these introductions!**

Each week holds 2 double or triple lectures covering the math-topics (possibly including on the spot theoretical training exercises) – **Attend these and read the book and eventually the lec. notes!**

Week 1-2 hold additionally 2 double lectures focused on Maple and week 5 an additional single lecture on Maple – **Attend these and read the lecture notes and/or the Maple manuals if you are not a fluent programmer (or followed the similar lectures in NumIntro)!**

Weeks 1, 3, 6 and 8 contain background lectures for the weekly assignments. **Attend these!**

Theory is not enough to learn the subjects. You must also experiment:

Each week holds 4 hours of exercise sections where you can get help from a teaching assistant.

For each week there is a compulsory weekly assignment. **The compulsory assignments must be handed in for grading by the instructor.**

Weekly assignments and final exam:

- 7 weekly compulsory assignments.
  The weekly assignments should be worked out and handed in by groups of 3 no later than midnight, Sunday of the week. For the 7th assignment, the deadline is by midnight, Friday of the week.
  Individual hand in's or groups of 2 or 4 is by permission only.
  It is a requirement to participate in the final exam, that the 7 weekly assignments are approved and valid Monday in week 8 of the block. (An assignment expires after Block 3 of the following year).
  If a weekly assignment is failed, it can be resubmitted once in a corrected version one week after the original hand in date. The 7th and last weekly assignment can be resubmitted only as part of the reexam. (But see "fast reexam" below).
- The final exam consists of a final project to be worked out in week 8-9 and handed in individually no later than midnight Sunday at the end of Block 3.
  The course is pass/fail based on the final exam except for exchange students that can demonstrate the need for a grade (see below).
- Reexam: Same as the final exam except that only one week (full time) is allowed for the final project that must be turned in no later than midnight Sunday at the end of Week 26.
  Those of the 7 weekly compulsory assignments that are not approved must be handed in no later than midnight Sunday at the end of Week 25.
- Fast reexam in block 3: If the first 6 weekly assignments are approved by Monday in week 8 of block 3 but the 7th weekly assignment is not approved by this deadline but is resubmitted no later than midnight, Sunday of week 8 of the course – and passes, then the final project will be accepted and graded together with the final projects for the ordinary exam.

A grade for an exchange student is given based on the week 2-7 assignments and the final project. Exchange students requiring a grade must ask the teachers to get a grade **and** hand in the weekly assignments for week 2-7 and the final project individually and the grade will be based on all of these.

Weekly plan for NumDiff, Block 3A, 2012 week 1-7:

Mon	Tuesday		Wed	Thursday		Fri	Sat	Sun
	8:15-9:00	**Intro** (Aud. 3 Aug. Krogh or* A107)		8:15-10:00	**Lecture 2** (Aud. 2 HCØ or* A107)			
	9:15-12:00	**Lecture 1*** (Aud. 3 Aug. Krogh or* A107) **Week 1, 3, 6 Case pres.**		10:15-12:00	**Maple in week 1, 2, 5 Selfstudy otherwise** (Aud. 2 HCØ or* A107)			
		Study Lec1 (after) + Study Lec2 (before)		12:15-13:00				
				13:15-17:00	**Weekly assignment with consultation bring laptop** (A107 and A110)			**Hand in last week's homework Before midnight**
					Finish weekly assignment			

* To be announced. Week 1 Aud. 3 Aug. Krogh and Aud. 2 HCØ will be used.

A 3 hour lecture typically consists of

1. Presentation of a topic
2. Classroom exercises on the topic
3. Small break

Repeat 1-3.

Weekly plan for NumDiff, Block 3A, 2012 week 8-9:

Monday in week 8	Tuesday in week 8 (27/3)		Wednesday in week 8	…	Sunday in week 9 (15/4)
	8:15-9:00	**Theory for final assignment** (A107 or* Aud. 3 Aug. Krogh)			
	9:15-10:00	**Case presentation** (A107 or* Aud. 3 Aug. Krogh)			
	10:15-11:00	**Talk about final assignment** (A107 or* Aud. 3 Aug. Krogh)			
Selfstudy		**Selfstudy**	**Selfstudy**	**Selfstudy**	**Selfstudy** **Hand in final assignment Before midnight**

Study program - NumDiff: Lectures and exercises

Week	Time	Name	Topic/Problems
6[1] 7+9/2	Tu 8^{15}-9	Week Intro [JH,JS,JP,SN]	1) JH, JS, JP, SN: Presentation of teachers. 2) JH: The course work- and examform. (NumDiff.pdf) 3) JH: Questions from students. 4) JS: This weeks material.
	Tu 9-11	Lec 1 [JS]	Pp.223-229 (App.A) a) Order symbol "Big Oh" incl. exercises. b) Taylor expansion incl. exercises. c) Round-Off error – the smile curve. No exercises.
	Tu 11-12	Week 1-2 [JH]	Background lecture for weekly assignments for week 1-2: *Jens Hugger, y'=Ay, matrix exponential and truncation error.*
	Th 8-10	Lec 2 [JS]	Pp. 1-8 (Ch.1) a) Differential equations – concepts, existence & uniqueness incl. exercises. b) 1-4 of the 5 steps for constructing a numerical algorithm incl. exercises for step 3: Difference formulas, truncation error and consistency.
	Th 10-12	Mtheory 1-2 [JH]	Basic Maple 1-2
	Th 13-17	Week 1 [JP,SN]	Weekly assignment
7[2] 14+16/2	Tu 8^{15}-9	Week Intro [JS,JP,SN]	1) JS: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JS: Last weeks material. 4) JS: This weeks material.
	Tu 9-11	Lec 3 [JS]	Pp.8-18 (Ch.1) a) Discretization and round-off error. Error measures. No exercises. b) Convergence: Handout for "Lax: A consistent method that is 0-stable (contd. dependence on data) is convergent. Incl. exercises. c) A-stability as "behavior for finite step sizes" incl. exercises. d) Examples of methods (table 1.3) incl. exercises.
	Tu 11-12	Week 2 [FL]	Background lecture for weekly assignment for week 2: *Francois Lauze, background for Euler-Lagrange eqns. in computer imaging.*
	Th 8-10	Lec 4 [JS]	Pp.18-33 (Ch.1) a) Quadrature and Runge-Kutta-methods as examples of table 1.3. b) Extensions and ghost points: Better understanding of step 3 and order of consistency depends also on the order of the boundary condition approx. c) Conservative methods: "Selecting methods not just for order and stability but to mimic special properties".
	Th 10-12	Mtheory 3-4 [JH]	Statements and Procedures
	Th 13-17	Week 2 [JP,SN]	Weekly assignment
8[3] 21+23/2	Tu 8^{15}-9	Week Intro [JS,JP,SN]	1) JS: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JS: Last weeks material. 4) JS: This weeks material.
	Tu 9-11	Lec 5 [JS]	Pp.45-58 (Ch.2) a) Examples of methods. b) Losing the arrow of time. c) Repeat: Step 1-4 in constructing an FDM. d) Tridiagonal matrices. e) Modelling and roundoff errors. f) Repeat: More complicated boundary conditions. g) Nonlinear problems – Newtons method.
	Tu 11-12	Week 3-5 [JH]	Background lecture for weekly assignments for week 3-5: *Jens Hugger, Testing and explaining errors*
	Th 8-10	Lec 6 [JS]	Pp.58-73 (Ch.2) a) Residual methods b) Homogenization c) Shooting methods – regaining arrow of time, transforming higher order DEP to system of first order DEP's.
	Th 10-12	Week 3 [ ]	Weekly assignment - Selfstudy
	Th 13-17	Week 3 [JP,SN]	Weekly assignment

9[4] 28/2+1/3	Tu 8^{15}-9	Week Intro [JS,JP,SN]	1) JS: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JS: Last weeks material. 4) JS: This weeks material.
	Tu 9-12	Lec 7 [JS]	Pp.83-100 (Ch.3) a) 2nd order, linear 2D PDE's. b) Classification of 2nd order, linear 2D PDE's. c) Properties of the exact solution to the heat equation. d) Step 1-4 for constructing a numerical algorithm for a PDE. e) Matrix formulation for explicit method. f) Stability and the connection to convergence.
	Th 8-10	Lec 8 [JS]	Pp.100-119 (Ch.3) a) Implicit method and matrix formulation. b) Crank-Nicolson method and L-stability. c) Dimension reduction (MOL) and Collocation. d) Nonlinear equations (Burger) in particular CN for nonlinear equations.
	Th 10-12	Week 4 [ ]	Weekly assignment - Selfstudy
	Th 13-17	Week 4 [JP,SN]	Weekly assignment
10[5] 6+8/3	Tu 8^{15}-9	Week Intro [JH,JS,JP,SN]	1) JS: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JS: Last weeks material. 4) JH: This weeks material.
	Tu 9-12	Lec 9 [JH]	Book pp.127-132 (Ch.4) – Lecture notes pp. 3.1, 4.1-4.3 a) Survey of classes of PDE's. b) Properties of the exact solution to the advection equation. c) Weak solutions
	Th 8-10	Lec 10 [JH]	Book pp.132-147 (Ch.4) – Lecture notes pp. 4.4-4.7 a) Examples of methods for advection. b) Truncation error for advection methods. c) Stability for advection methods. d) CFL and monotonicity.
	Th 10-11	Mtheory 5 [JH]	Input/output
	Th 11-12	Week 5 [ ]	Weekly assignment - Selfstudy
	Th 13-17	Week 5 [JP,SN]	Weekly assignment
11[6] 13+15/3	Tu 8^{15}-9	Week Intro [JH,JP,SN]	1) JH: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JH: Last weeks material. 4) JH: This weeks material.
	Tu 9-11	Lec 11 [JH]	Book pp.155-164 (Ch.5) – Lecture notes pp. 5.1-5.5 a) Properties of the exact solution to the wave equation.
	Tu 11-12	Week 6-7 [RP]	Background lecture for weekly assignments for week 6-7: *Rolf Poulsen, Solving financial derivatives*
	Th 8-10	Lec 12 [JH]	Book pp.164-176 (Ch.5) – Lecture notes pp. 5.6-5.8 a) Examples of methods for waves. b) Stability for wave methods. d) Practical behavior of wave methods.
	Th 10-12	Week 6 [ ]	Weekly assignment - Selfstudy
	Th 13-17	Week 6 [JP,SN]	Weekly assignment
12[7] 20+22/3	Tu 8^{15}-9	Week Intro [JH,JP,SN]	1) JH: Questions from students. 2) JP, SN: General problems with last weeks assignment. 3) JH: Last weeks material. 4) JH: This weeks material.
	Tu 9-12	Lec 13 [JH]	Book pp.181-192 (Ch.6) – Lecture notes pp. 6.1-6.4 a) Properties of the exact solution to the Laplace equation. b) A method for Laplace's equation. c) The Laplace matrix.
	Th 8-10	Lec 13 (excl. 13.2, 13.3) Lec 14 (excl. 14.3, 14.4)	Book pp.192-214 (Ch.6) – Lecture notes pp. 6.5, 6.8, 6.9, 6.10 ~~a) Properties of the Laplace matrix.~~ (Book § 6.2.2 from TEST 1) b) Iterative metods for solving the Laplace equation system – Descent methods c) Steepest descent method.

		[JH]	d) Conjugate gradient method. ~~e) Preconditioned conjugate gradient method.~~ (Book § 6.5)
	Th 10-12	Week 7 [ ]	Weekly assignment - Selfstudy
	Th 13-17	Week 7 [JP,SN]	Weekly assignment
13[8] 27/3	Tu 8^{15}-9	Week 8-9 [JH]	Background lecture for final project for week 8-9: *Jens Hugger, Crank Nicolson for nonlinear problems*
	Tu 9-10	Week 8-9 [PGS]	Background lecture for final project for week 8-9: *Preben Graa Sørensen, Patterns on snail shells*
	Tu 10-11	Week 8-9 [JP,SN]	Final assignment
	-	-	Questions by e-mail or appointment
14-15[9]	-	-	Questions by e-mail or appointment