Aim

The purpose of Holomorphic Day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant DFF|181-00502 from The Danish Council for Independent Research | Natural Sciences.

Program

The talks take place in Auditorium 10 at the H.C. Ørsted Institute. The auditorium is near the department of Mathematical Sciences and is located on the first floor of the long walking area in the H.C. Ørsted Institute.

Coffee is served from 9:30 outside the auditorium. We take lunch in a self-service restaurant on campus. We propose to go there together.

A common dinner at a nearby restaurant is planned after the talks.

Each talk is scheduled for 50 minutes plus questions. We aim for a short intermission between the talks.

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Abstracts

Straightening the square

ARNAUD CHÉRITAT
TOULOUSE, FRANCE

One interesting and very useful way of creating deformations in the plane is by "straightening ellipse fields". An ellipse field records how a function distorts angles at small scales; a straightening is a map from the plane to the plane solves a simple partial differential equation: namely, all ellipses in the field must be mapped to circles. When it exists, the straightening map is essentially unique and can be used to solve various problems in geometry and topology.

We will look at a deceptively simple situation: outside a given square the ellipse field consists of circles and inside the square the ellipses are all parallel to the vertical side, with fixed ratio $K = \text{major axis} / \text{minor axis}$. In this case, there is a straightening map defined on the plane and the image of the square under the map is quite interesting! Now, here is a puzzle: what happens to this shape as $K$ tends to infinity?

The “hole story” of a forgotten function

DARREN CROWDY

IMPERIAL COLLEGE, ENGLAND

Motivated by problems arising in the applied sciences, this talk surveys a new theoretical approach to solving problems in multiply connected planar domains as developed by the speaker (and his group) in recent years. Multiply connected domains are ubiquitous in applications; whenever two or more objects/entities interact in some ambient medium the analysis may call for the methods discussed in this lecture.

We will advocate the use of a holomorphic function known as “Schottky-Klein prime function” – a very important classical special function that is hardly known to non-specialists but which is relevant to a surprisingly wide range of mathematical problems often facilitating concise and elegant representations of their solutions.

Some illustrative example problems from applications will be described and their solutions explicitly constructed. We will also describe freely available numerical codes that we have developed for the computation of the prime function in order to promote its use.

We hope to demonstrate that the new methods are sufficiently general that they provide broad scope for tackling a variety of mathematical problems.
Manin, Mumford and Hénon

Romain Dujardin

UPMC, France

The dynamical Manin-Mumford problem aims at understanding under which conditions an algebraic dynamical systems can have “unusually many” periodic points on a subvariety. This is motivated by classical questions in arithmetic geometry. In the talk, I will discuss this problem in the setting of polynomial automorphisms of $\mathbb{C}^2$. This is joint work with Charles Favre (Ecole Polytechnique).

Dynamics of orthogonal polynomials

Carsten Lunde Petersen

Roskilde University, Denmark

The research fields of orthogonal polynomials on the one side and holomorphic dynamics on the other side do not have much prior interaction. But is there anything we can learn about holomorphic dynamics from the theory of orthogonal polynomials? And vice versa? In a recent paper we have investigated these questions and come up with surprising answers. In the talk I will report on this work and on further results.

Joint work with Jacob Storald Christiansen, Christian Henriksen and Henrik Laurberg Pedersen.

Positive linear maps and rigid functions (Choi’s conjecture)

Mitsuru Uchiyama

Shimane University, Japan

Let a linear map $\Phi$ between two unital $C^*$-algebras be positive and preserve the identity. Kadison showed that if $f(t) = |t|$ and $\Phi(f(A)) = f(\Phi(A))$ for all selfadjoint operators $A \in A$, then $\Phi(A^2) = \Phi(A)^2$ for all selfadjoint operators $A$, that is, $\Phi$ is a $C^*$-homomorphism. Choi proved this fact for an operator convex function $f$, and then conjectured that this fact would hold for a non-affine continuous function $f$. We shall prove a
refinement of his conjecture. Petz has further proved that if \( f(\Phi(A)) = \Phi(f(A)) \) for a non-affine operator convex function \( f \) and a fixed \( A \), then \( \Phi(A^2) = \Phi(A)^2 \). Arveson called such a function \( f \) a rigid function. We shall directly show power functions \( t^r \) are rigid functions on \((0, \infty)\) if \( r \neq 0, r \neq 1 \).