Holomorphic Day, April 15, 2011
Department of Mathematical Sciences
University of Copenhagen

Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 10-083122 from The Danish Council for Independent Research | Natural Sciences

1 Schedule

The lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.45-10.15 in E 419 (Fourth floor)

Jacob Stordal Christiansen: 10.15-11.05 in Aud. 10
Orthogonal polynomials on finite and infinite gap sets

Michel Zinsmeister: 11.15-12.05 in Aud. 10
Bieberbach coefficients for SLE (Stochastic Loewner Equation)

Lunch: 12.15-13.15

Alexander Rashkovskii: 13.15-14.05 in Aud. 10
Classical and new loglog-theorems

Kristian Seip: 14.15-15.05 in Aud. 10
The Bohnenblust-Hille inequality for homogeneous polynomials

Coffee break: 15.05-15.30

Kari Astala: 15.30-16.20 in Aud. 10
Random conformal welding

Asli Deniz: 16.30-17.20 in Aud. 10
A landing theorem for periodic rays for entire transcendental maps with bounded post-singular set.

Dinner: 18.30-

2 Abstracts

Jacob Stordal Christiansen, Steno Research Fellow, University of Copenhagen: *Orthogonal polynomials on finite and infinite gap sets*

The theory of orthogonal polynomials on finite unions of compact intervals can be generalized to infinite gap sets of Parreau–Widom type. This notion of regular compact sets includes Cantor sets of positive measure, among others.

When E is a Parreau–Widom set of positive measure, the equilibrium measure $d\mu_{\mathsf{E}}$ of E is absolutely continuous. By rescaling, if necessary, we may assume that the logarithmic capacity of E is 1. For probability measures $d\mu = f(t)dt + d\mu_{\mathrm{s}}$ with essential support E, we shall concentrate on the Szegő condition

$$\int_{\mathsf{E}} \log f(t)\, d\mu_{\mathsf{E}}(t) > -\infty. \tag{$*$}$$

Under certain assumptions on the mass points of $d\mu_{\mathrm{s}}$ outside E, we show that $(*)$ is equivalent to boundedness of the leading coefficients in the associated orthonormal polynomials P_n.

The question of polynomial asymptotics will also be discussed. In particular, we investigate to which extent P_n admits a power asymptotic behavior (aka Szegő asymptotics). In this connection, the isospectral torus $\mathcal{T}_{\mathsf{E}}$ of dimension equal to the number of gaps in E will be introduced as the key player. Our analysis relies on the covering space formalism introduced into spectral theory by Sodin–Yuditskii. This allows us to lift functions on the multiply connected domain $\overline{\mathbb{C}} \setminus \mathsf{E}$ to the unit disk $\mathbb{D}$. The universal covering map $\mathbf{x} : \mathbb{D} \to \overline{\mathbb{C}} \setminus \mathsf{E}$ is linked with a Fuchsian group Γ of Möbius transformations in such a way that

$$\mathbf{x}(z) = \mathbf{x}(w) \iff \exists \gamma \in \Gamma : \; z = \gamma(w).$$

As we shall see, it is crucial whether or not the Abel map from $\mathcal{T}_{\mathsf{E}}$ to Γ^*, the multiplicative group of characters of Γ, is a homeomorphism.

Michel Zinsmeister, Professor, University of Orléans: *Bieberbach coefficients for SLE (Stochastic Loewner Equation)*

In 1923 Loewner proved Bieberbach's conjecture for $n = 3$; he found for this purpose a very general method describing the growth of slit domains with the aid of a continuous function from $\mathbb{R}_+$ into the circle . Remarkably, the process can be reversed: he showed indeed that every such function gives rise to a growth process. In 1999 Oded Schramm revived this theory by taking as such driving function a constant time a one dimensional brownian motion, giving rise to the celebrated SLE theory. The aim of this talk is to revisit Bieberbach conjecture in the framework of whole-plane SLE.

Alexander Rashkovskii, Professor, University of Stavanger: *Classical and new loglog-theorems.*

Celebrated loglog-theorems of Carleman, Wolf, Levinson, Sjöberg give uniform bounds on analytic functions whose majorants are double logarithmically integrable. We present a unified approach to such theorems and, moreover, obtain stronger results by replacing the original pointwise bounds with integral ones. A typical statement is that if the integral of a nonnegative subharmonic function with respect to a certain measure on the circle of radius R has the upper bound $V(R)$, then the function has the pointwise upper bound $cV(bR)$ for some constants c and b. We show that a famous theorem due to Matsaev on entire functions with special lower bound can be treated in the same way as well. The main ingredient is a simple description for radial projections of harmonic measures of bounded star-shaped domains in the plane, which, in particular, explains where the loglog-conditions come from. This can be of interest in other questions, where functions of this kind appear.

Kristian Seip, Professor, Norwegian University of Science and Technology: *The Bohnenblust-Hille inequality for homogeneous polynomials*

The Bohnenblust-Hille inequality says that the $\ell^{\frac{2m}{m+1}}$-norm of the coefficients of an m-homogeneous polynomial P on $\mathbb{C}^n$ is bounded by $\|P\|_\infty$ times a constant independent of n, where $\|\cdot\|_\infty$ denotes the supremum norm on the polydisc $\mathbb{D}^n$. In a recent paper with A. Defant, L. Frerick, J. Ortega-Cerdà, and M. Ounaïes, we prove that this inequality is hypercontractive, i.e., the constant can be taken to be C^m for some $C > 1$. We discuss the background for this result, beginning with Harald Bohr's absolute convergence problem for Dirichlet series, as well as some consequences, including a remarkable asymptotic formula for the Sidon constant for Dirichlet polynomials.

Kari Astala, Professor, University of Helsinki: *Random conformal welding*

One of the recent success stories of the methods of complex analysis is the description of random curves that arise as scaling limits of different lattice models of statistical physics, such as percolation or the ising model. Here the Stochastic (or Schramm) Loewner equation (SLE) has been particularly powerful.

In this talk, based on joint work with Peter Jones, Antti Kupiainen and Eero Saksman, we will consider an approach to producing random Jordan curves, based on the method of conformal welding. This approach provides a natural correspondence between (certain) Jordan curves in the complex plane and (suitable) homeomorphisms of the unit circle. Using this idea we construct random curves from random circle homeomorphisms, whose derivative is proportional to the exponential of the Gaussian free field, or more precisely, the restriction of the two dimensional free field to the circle.

Asli Deniz, phd-student, University of Barcelona: *A landing theorem for periodic rays for entire transcendental maps with bounded post-singular set.*

We investigate some landing properties of dynamic rays for entire transcendental maps of finite order or finite composition of finite order maps, which have bounded post-singular set. For this class, we give an alternative proof to show all periodic dynamic rays land, by using standard hyperbolic geometry results.

Organized by **Christian Berg, Christian Henriksen, Henrik Laurberg Pedersen, Carsten Lunde Petersen**