

**BANACH ALGEBRAS AND
LOCAL SPECTRAL THEORY**

**CONFERENCE
IN HONOUR OF
KJELD BAGGER LAURSEN**

**DEPARTMENT OF MATHEMATICAL SCIENCES
UNIVERSITY OF COPENHAGEN
AUGUST 28-30 2008**

ABSTRACTS

Pietro Aiena. *A variant of Weyl's theorem*

Property (*w*) for linear bounded operators on Banach spaces is a variant of Weyl's theorem. In this talk we discuss the preservation of this property under some classes of commuting perturbations.

Ernst Albrecht. *Chebyshev polynomials for linear operators*

After a short survey on the history of Chebyshev polynomials we consider the problem of uniqueness for Chebyshev polynomials for linear operators and elements of Banach algebras.

Janko Bračič. *Local spectra of a decomposable multiplication operator*

Let A be a unital semisimple commutative complex Banach algebra. If $a \in A$ induces a decomposable multiplication operator T_a on A , then the local spectra of T_a are given by $\sigma_{T_a}(x) = \widehat{a}(\text{supp } \widehat{x})$ ($x \in A$). First we observe that the support $\text{supp } \widehat{x}$ can be replaced by the Beurling spectrum $sp(x)$ of $x \in A$. Then we show that the equalities similar to $\sigma_{T_a}(x) = \widehat{a}(sp(x))$ ($x \in A$) hold also for some $a \in A$ when the semisimplicity of A is not assumed.

Eggert Briem. *A Stone-Weierstrass property for Banach function spaces*

We consider a strong lattice property for a Banach function space B on a compact Hausdorff space, which gives a general Stone-Weierstrass theorem for B . We also study the relation of this theorem and its proof to a certain decomposition of an associated compactification, and to another lattice-like property.

H. Garth Dales. *Weighted convolution algebras*

My one joint paper with Kjeld Bagger Laursen (also with Bill Bade) was a memoir on Banach algebras of power series, aka weighted convolution algebras on the natural numbers. I will recall briefly some results from that memoir, and then consider similar algebras in a more general setting : these are weighted convolution algebras on subsemigroups of $(\mathbb{R}, +)$, especially on the rational numbers. In particular I shall talk about when they are Arens regular and when they are strongly Arens irregular and when they are neither; some of the latter results are joint work with Haresh Dedania.

Joerg Eschmeier. *Fredholm theory and Samuel multiplicity*

Let T be a Fredholm operator on a Banach space X . The dimensions of the kernels and cokernels of the powers T^k of T are easily seen to grow at most like a linear polynomial in k . It was observed by Xiang Fang that the first order coefficients of the corresponding polynomials are the constant values of $\dim \text{Ker}(z - T)$ and $\dim X / \text{Im}(z - T)$ on suitable pointed neighbourhoods of $z = 0$. We shall report on some recent multivariable generalizations of this result.

Niels Grønbaek. *Homological triviality of semigroup algebras*

Let S be a semigroup and let $\ell^1(S)$ be its discrete convolution algebra. In trying to understand the structural implications of homological triviality of Banach algebras, these algebras have been in focus. In particular amenability has been characterized in terms of the underlying semigroup. Recently other notions of homological triviality, biflatness and biprojectivity, have been characterized for S a semilattice by Yemon Choi. In the talk I will give a brief overview and present a complete characterization of biflatness and biprojectivity for abelian semigroups. This is joint work with F. Habibian.

Robert Kantowitz. *Approximation by operators that preserve disjointness*

This talk, based on joint work with Michael M. Neumann, begins with a discussion of approximation of continuous linear functionals on the algebra $C(X)$, where X is compact Hausdorff, by functionals that preserve disjointness. With a second compact Hausdorff space Y , we then move into the more general situation of approximating arbitrary continuous linear mappings from $C(X)$ into $C(Y)$ by disjointness preserving operators, which, in this context, are known to be weighted composition operators.

Niels Jakob Laustsen. *Commutation relations in Banach algebras*

Since the mathematical formulation of Heisenberg's Uncertainty Principle, commutators have played an important role in the study of non-commutative rings, including Banach algebras. A celebrated classical theorem of Wintner and Wielandt states that it is impossible to express the identity element of a unital Banach algebra as a commutator, thus effectively ending any hope that quantum mechanics can be formalized mathematically in a Banach-algebra setting.

Nevertheless, there is continuing interest in realizing various other commutation relations in Banach algebras. Following a question of S. Silvestrov, we have constructed a family of Banach algebras in which the so-called (q_1, q_2) -deformed Heisenberg–Lie commutation relations (where q_1 and q_2 are non-zero complex numbers) are realized in a universal way.

In the talk I shall explain this result together with its $*$ -algebraic counterpart. This is joint work with S. Silvestrov.

Martin Mathieu. *What can the spectrum tell us?*

Len Miller. *The closed range property and spectral inclusions for Banach space operators*

Let T be a bounded operator on a complex Banach space X , and let $H(V, X)$ denote the Fréchet space of analytic X -valued functions on an open subset V of the complex plane. We give a condition which implies that the mapping $T_V f(z) := (T - z)f(z)$ has closed range in $H(V, X)$. Moreover, we show that there is a largest open set U for which the map T_V has closed range in $H(V, X)$ for all $V \subseteq U$. As an application, we show that this closed range condition and its weak- $*$ counterpart play key roles in the relations of parts of the spectra of two (asymptotically) intertwined operators.

Michael Neumann. *Bounded local resolvent functions*

This talk centers around a new method of constructing bounded local resolvent functions for continuous linear operators on Banach spaces with applications to convolution operators on the group algebra of a locally compact abelian group. The general approach utilizes the concept of algebraic spectral subspaces in the spirit of the work of Kjeld Laursen. The talk is based on recent collaboration with Vladimir Mueller.

Werner J. Ricker. *The Hausdorff-Young inequality*

In 1967 R.E. Edwards posed the following question. Let $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$ be the circle group and $1 < p < 2$. Which functions $f \in L^1(\mathbb{T})$ have the property that their Fourier transform satisfies

$$(\chi_A f)^\wedge \in \ell^{p'}(\mathbb{Z}), \quad \forall \text{ Borel sets } A \subseteq \mathbb{T},$$

where $\frac{1}{p} + \frac{1}{p'} = 1$? The Hausdorff-Young inequality ensures that every $f \in L^p(\mathbb{T})$ satisfies the stated condition. A complete solution to the above question will be presented.

Wieslaw Zelazko. *What I cannot prove about ideals in F -algebras*