1. Let $\mathcal{A}$ be a finite alphabet. Show that the map $d : \mathcal{A}^{\mathbb{Z}} \times \mathcal{A}^{\mathbb{Z}} \longrightarrow [0, \infty[$ defined by

$$d(x,y) = \begin{cases} 2^{-n} & \text{if } x \neq y \text{ and } n \text{ is maximal with } x_{[-n,n]} = y_{[-n,n]} \\ 0 & \text{if } x = y \end{cases}$$

is a metric on $\mathcal{A}^{\mathbb{Z}}$. Prove that it is equivalent with the metric

$$d'(x,y) = \sum_{n \in \mathbb{Z}} 2^{-|n|} d_{\mathcal{A}}(x_n, y_n)$$

where $d_{\mathcal{A}}$ is the discrete metric on $\mathcal{A}$.

2. Prove that $x^{(i)} \longrightarrow x$ in $(\mathcal{A}^{\mathbb{Z}}, d)$ exactly if

$$\forall n \in \mathbb{Z} \exists i_0 \in \mathbb{N} : i \geq i_0 \Longrightarrow x_n^{(i)} = x_n$$

3. Prove that when the shift spaces X and Y are equipped with metrics as above, then any sliding block code $\phi : X \longrightarrow Y$ is continuous.

4. Let $\mathcal{A}$ be a finite alphabet. Prove that $X \subseteq \mathcal{A}^{\mathbb{Z}}$ is a shift space precisely when it satisfies

   (i) X is shift invariant

   (ii) X is closed in $(\mathcal{A}^{\mathbb{Z}}, d)$.

5. Let X and Y be shift spaces. Prove that $\phi : X \longrightarrow Y$ is a sliding block code precisely when it satisfies

   (i) ϕ is stationary (i.e. $\phi \circ \sigma = \sigma \circ \phi$)

   (ii) ϕ is continuous as a map from (X, d) to (Y, d).

6. Give new proofs of LM 1.5.13 and 1.5.14 based on exercise 4 and 5 above.