```
restart;
alpha:=sqrt(2)-1;
```

$$\alpha := \sqrt{2} - 1 \tag{1}$$

```
with(StringTools):
MakeString:=(a,NN)->Implode([seq(convert(floor((n+1)*a)-floor(n*a),string),n=0..NN)]);
```

$$MakeString := (a, NN) \to StringTools:-Implode([seq(convert(floor((n+1)\cdot a) - floor(n\cdot a), string), n = 0..NN)]) \tag{2}$$

```
s:=MakeString(alpha,50);
```

$$s := "00101001010010101001010010101001010010101001010100101" \tag{3}$$

```
seq(convert(NGrams(s,n),set),n=1..5);
```

$$\{"0", "1"\}, \{"00", "01", "10"\}, \{"001", "010", "100", "101"\}, \{"0010", "0100", "0101", "1001", "1010"\}, \{"00101", "01001", "01010", "10010", "10100", "10101"\} \tag{4}$$

```
getDiv:=proc(i,j) local T; T:=[seq(frac(frac(n*evalf(alpha))+1),n=i..j),1]; T:=sort(T); return T; end;
```

$$getDiv := \mathbf{proc}(i, j) \tag{5}$$

```
    local T;
    T := [seq(frac(frac(n*evalf(alpha))^1), n = i..j), 1]; T := sort(T); return T
end proc
```

```
with(plots): with(plottools):
```

```
getColor:=proc(i,N) if (i mod 2=1) then return COLOR(HUE,i/(2*N)); else return COLOR(HUE,1-i/(2*N)); end; end;
```

$$getColor := \mathbf{proc}(i, N) \tag{6}$$

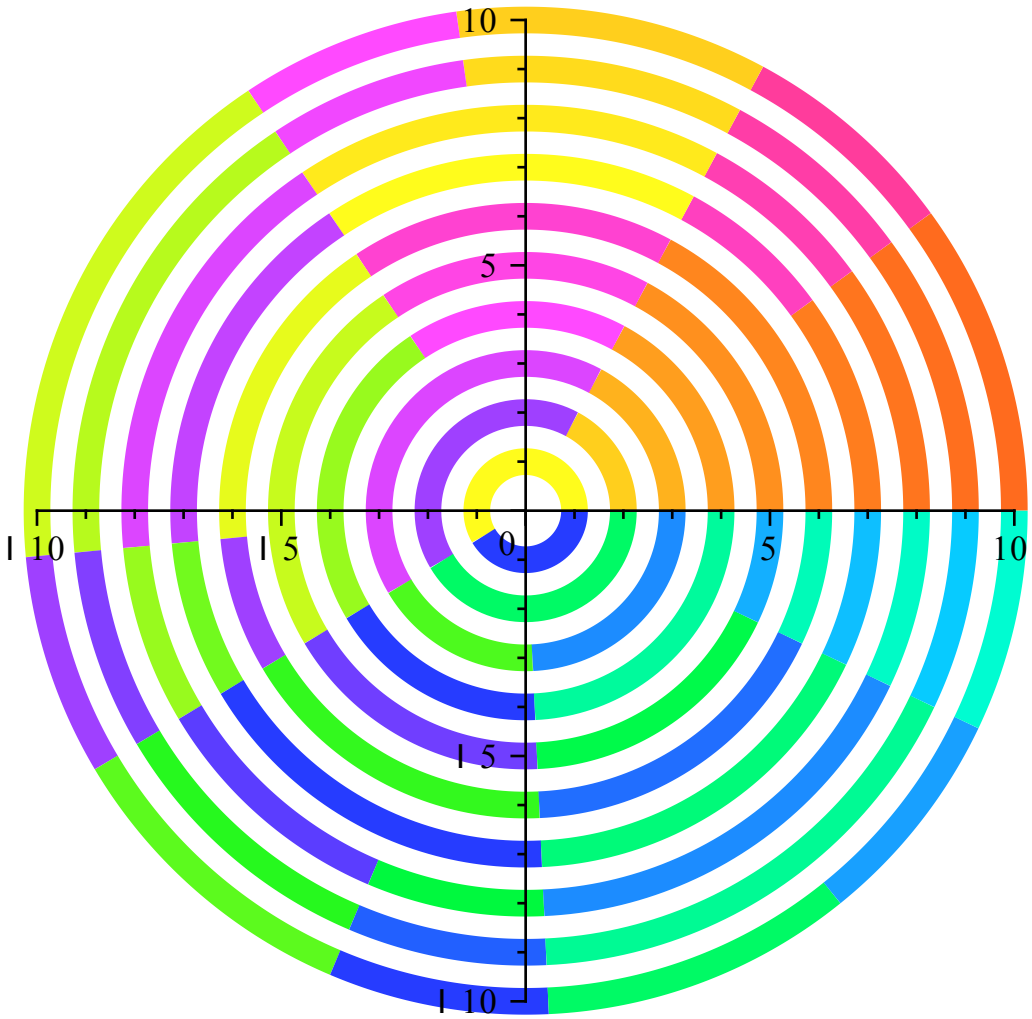
```
    if mod(i, 2) = 1 then
        return COLOR(HUE, 1/2*i/N)
    else
        return COLOR(HUE, 1-1/2*i/N)
    end if
end proc
```

```
plotMe:=proc(T,r) display([seq(arc([0,0],r,2*Pi*T[i]..2*Pi*T[i+1],color=getColor(i,nops(T)),thickness=10),i=1..nops(T)-1)]); end;
```

$$plotMe := \mathbf{proc}(T, r) \tag{7}$$

```
    plots:-display([seq(plottools:-arc([0, 0], r, 2*Pi*T[i]..2*Pi*T[i^1], color
     = getColor(i, nops(T)), thickness = 10), i = 1..nops(T)-1)])
end proc
```

```
display(seq(plotMe(getDiv(-n,0),n),n=1..10));
```




```
M  s; SubstituteAll(s,"0","01"); SubstituteAll(s,"1","01");
```

"00101001010010101001010010101001010010100101010100101"

"0101101101011011010110110110101101101011011010110110101101101011011010\
11011"

"000100100010010001001001000100100010010001001000100100010010001001" **(8)**

```
M  convert(NGrams(SubstituteAll(s,"0","01"),6),set); convert(NGrams
   (SubstituteAll(s,"1","01"),6),set);
```

{"010110", "011010", "011011", "101011", "101101", "110101", "110110"}

{"000100", "001000", "001001", "010001", "010010", "100010", "100100"} **(9)**

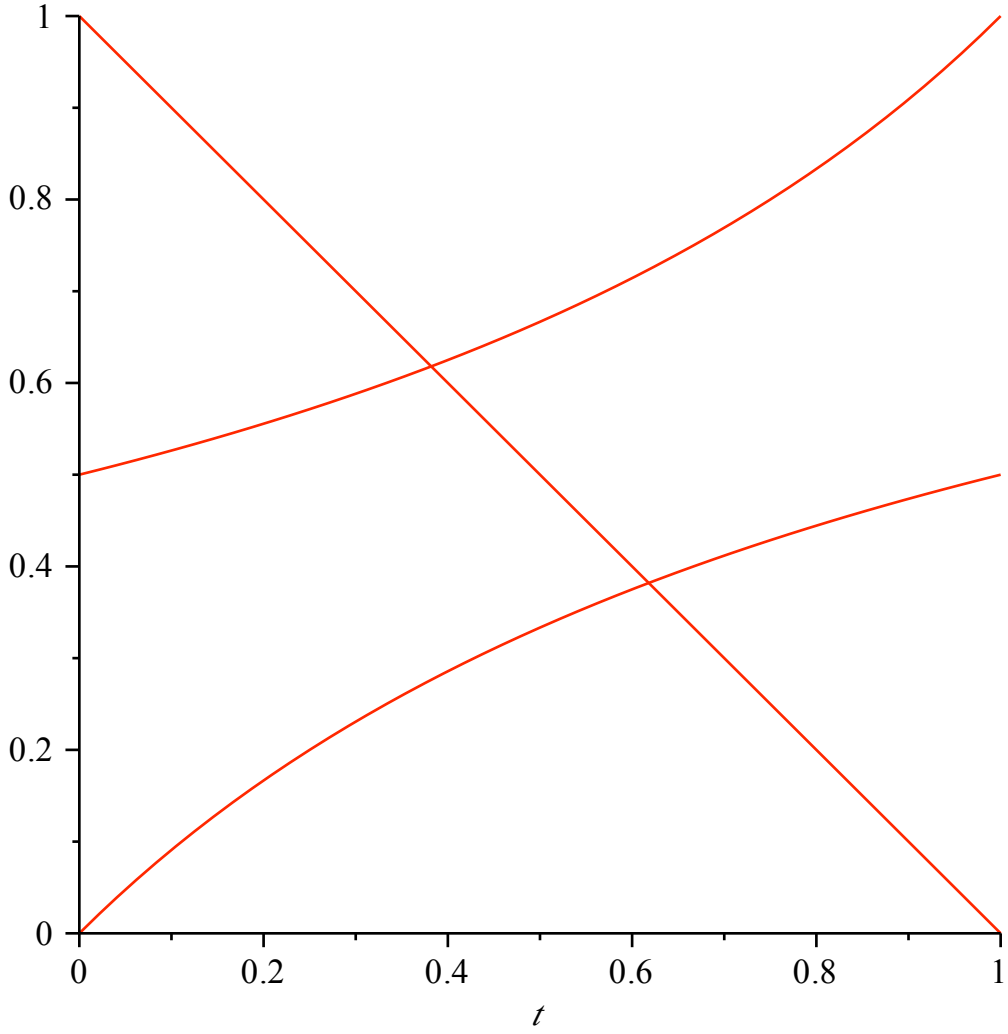
```
M  MakeString(alpha/(1+alpha),50);
```

"00010010001001000100100010010001001001000100100" **(10)**

```
M  MakeString(1/(2-alpha),50);
```

"01011011010110110101101101101011011010110110101011" **(11)**

```
M  display(plot(t/(1+t),t=0..1),plot(1/(2-t),t=0..1),plot(1-t,t=0.
   .1));
```




```
M with(numtheory):
M cfrac(alpha,10);
```

$$\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\cfrac{1}{2A+\frac{1}{2A+\cdots}}}}}}}}}}} \tag{12}$$

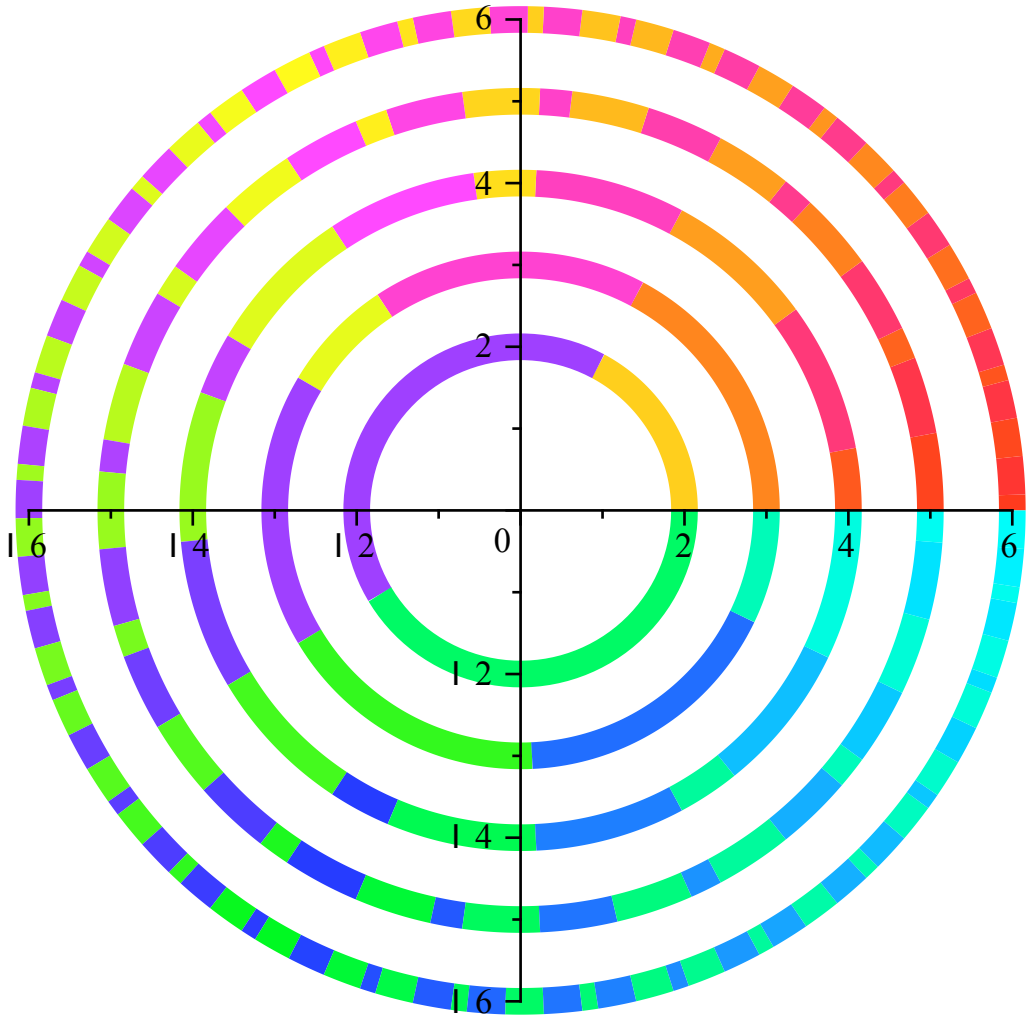
```
M pp:=seq(nthnumer(cfrac(alpha),n),n=0..10);
```

$$pp := 0, 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378 \tag{13}$$

```
qq:=seq(nthdenom(cfrac(alpha),n),n=0..10);
```

$$qq := 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378, 5741 \quad \textbf{(14)}$$

```
display(seq(plotMe(getDiv(-qq[n],qq[n-1]-1),n),n=2..6));
```

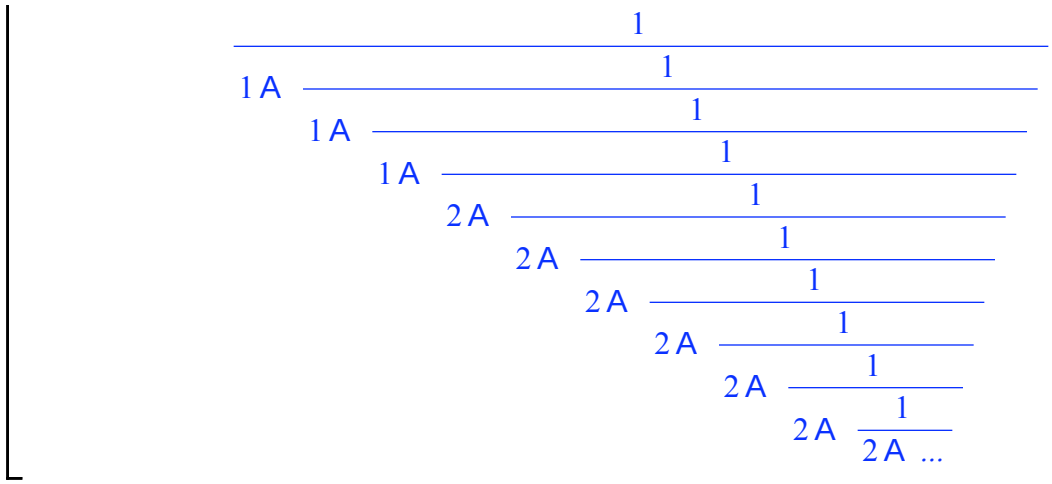



```
cfrac(alpha/(1+alpha),10);
```

$$\cfrac{1}{3A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \cfrac{1}{2A - \ldots}}}}}}}}}}} \quad \textbf{(15)}$$

```
cfrac(1/(2-alpha),10);
```

(16)



(16)