Bachelor projects for mathematics and mathematics-economics

Department of Mathematical Sciences
University of Copenhagen

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Contents

Introduction

This is a catalogue of projects suggested by the researchers at the Department for Mathematical Sciences for students in the B.S. programs in mathematics and mathematics-economics. It is important to note that such a catalogue will never exhaust all possibilities – indeed, if you are not finding what you are looking for you are strongly encouraged to ask the member of our staff you think is best qualified to help you on your way for suggestions of how to complement what this catalogue contains. Also, the mathematics-economics students are encouraged to study the searchable list of potenial advisors at the Economy Department on

`www.econ.ku.dk/polit/undervisning_og_opgaver/speciale/vejlederoversigt/`.

If you do not know what person to approach at the Department of Mathematical Sciences, you are welcome to try to ask the director of studies (Ernst Hansen, erhansen@math.ku.dk) or the assocate chair for education (Søren Eilers, eilers@math.ku.dk).

When you have found an advisor and agreed on a project, you must produce a contract (your advisor will know how this is done), which must then be approved by the director of studies at the latest during the first week of a block. The project must be handed in during the 7th week of the following block, and an oral defense will take place during the ninth week.

We wish you a succesful and engaging project period!

Best regards,

Søren Eilers	Ernst Hansen
Associate chair	Director of studies

1 Finance

1.1 Rolf Poulsen

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Relevant interests:

Finance.

Suggested projects:

- Option pricing [Fin1]
  Pricing and hedging of exotic options (barrier, American, cliquet). A detailed investigation of convergence in of the binomial model. Multi-dimensional lattices. Model calibration as an inverse problem.

- Stochastic interest rates [Fin1]
  Yield curve estimation. Estimation of dynamic short rate models. Calibration and the forward algorithm. Derivative pricing with applications to embedded options in mortgage products, the leveling algorithm. Risk management for mortgagors and pension funds.

- Optimal portfolio choice [Fin1]
  Quadratic optimization with linear but non-trivial constraints. Multi-period optimal portfolio choice via stochastic programming. An experimental approach to equilibrium.

- Model risk [Fin1]
  http://www.math.ku.dk/~rolf/Fin2_2010.doc

Previous projects:

- Pricing of cliquet option [Fin1]
- The Black-Litterman Model [Fin1]
- The critical line algorithm and beyond [Fin1]
- Financial networks and systemic risk [Fin1]

1.2 Other projects

Other projects in this area can be found with

- Jens Hugger (4.3)

2 Operations research

2.1 Louise Kallehauge

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Relevant interests:

Linear programming. Dynamic programming. Optimization. Modeling industrial problems, e.g. airline revenue management, liner shipping, vehicle routing.

Suggested projects:

- Convex analysis and minimization algorithms [OR1 (and Advanced OR)]
  When decomposing an integer programming problem, one is primarily minimizing a certain convex function called the dual function. The goal of the project is to solve the dual problem using an efficient algorithm. Various dual algorithms can be implemented and tested.

- Dynamic programming models vs. heuristics in airline revenue management [OR1 (and Advanced OR)]
  Analysis of which solution method is more suitable when assuming different arrival distributions for airline passengers. Implementation of a simple heuristic and possibly a dynamic programming model in e.g. Java, C++, or Visual Basic.

- Overbooking in airline revenue management [OR1 (and Advanced OR)]
  Extension of an existing dynamic programming model to include overbooking of flights in the airline industry. Implementation of the algorithm in e.g. Java, C++, or Visual Basic.

- Application of vehicle routing algorithms [OR1 (and Advanced OR)]
  The well-known vehicle routing problem has many applications in industrial contexts. Modeling a specific problem and solving it by standard methods, e.g. using GAMS and Cplex. Possibly implementation of a better solution algorithm in another programming language.

Previous projects:

- Overbooking in airline fenceless seat allocation [OR1, Advanced OR]

- Allocation of endangered species in European zoological gardens [OR1, Advanced OR]

- Implementation of a tabu search algorithm to solve the vehicle routing problem with time windows [OR1, Advanced OR]

- Scheduling of courses at the Department of Mathematical Sciences [OR1, Advanced OR]
- Analysis of different arrival distritubions and solution methods in airline revenue management [OR1, Advanced OR]

3 Algebra and number theory

3.1 Christian U. Jensen

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Relevant interests:

Galois theory. Algebraic number theory.

Suggested projects:

- Introductory Galois theory [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics, but is also a very rich field that can be studied in its own right.

- Introduction to algebraic number theory [Alg2]
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

3.2 Ian Kiming

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Relevant interests:

Algebraic number theory and arithmetic geometry.

Suggested projects:

- Introduction to algebraic number theory [Alg2]
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

- First case of Fermat's last theorem for regular exponents [Alg2]

The project studies the proof of Fermat's last theorem for 'regular' prime exponents p in the so-called first case: this is the statement that $x^p + y^p + z^p = 0$ does not have any solutions in integers x, y, z not divisible by p. The project involves studying some introductory algebraic number theory which will then also reveal the definition of 'regular primes'.

- p-ADIC NUMBERS [Alg2]
  The real numbers arise from the rational numbers by a process called 'completion'. It turns out that the rational numbers (and more generally any algebraic number field) has infinitely many other 'completions', namely one associated to each prime number p. The fields that arise in this way are called the fields of p-adic numbers. They have a lot of applications in many branches of mathematics, not least in the theory of Diophantine equations, i.e., the question of solving in integers polynomial equations with integral coefficients.

- HASSE–MINKOWSKI'S THEOREM ON RATIONAL QUADRATIC FORMS [Alg2]
  A rational quadratic form is a homogeneous polynomial with rational coefficients. The Hasse–Minkowski theorem states that such a polynomial has a non-trivial rational zero if and only if it has a non-trivial zero in the real numbers and in all fields of p-adic numbers. The latter condition can be translated into a finite number of congruence conditions modulo certain prime powers and thus one obtains an effective criterion. The project involves an initial study of p-adic numbers.

- CONTINUED FRACTIONS AND PELL'S EQUATION [Alg2]
  The project studies the theory of continued fractions and how this can be applied to determining units in quadratic number rings. This has applications to the study of Pell (and 'non-Pell') equations, i.e., solving equations $x^2 - Dy^2 = \pm 1$ in integers for a given positive, squarefree integer D.

- CLASS GROUPS OF QUADRATIC NUMBER FIELDS AND BINARY QUADRATIC FORMS [Alg2]
  A quadratic number field is a field obtained from $\mathbb{Q}$ by adjoining a number of form $\sqrt{D}$ where D is an integer that is not a square (in $\mathbb{Z}$.) The class group attached to such a field measures how far its so-called ring of integers in from being a unique factorization domain. These class groups are necessary to study of one wants to understand integer solutions to equations of form $ax^2 + by^2 = c$ for given integers a, b, c.

- MODULAR FORMS ON $\mathrm{SL}_2(\mathbb{Z})$ [Alg2, KomAn]
  This project studies modular forms on $\mathrm{SL}_2(\mathbb{Z})$. These are initially analytic objects and thus a certain, minimal background in complex analysis is required. Modular forms turn out to have a lot of deep connections to arithmetic, and one can use this project as a platform for a later study of the more general modular forms on congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$. These are very important in modern number theory and are for instance central in Andrew Wiles' proof of Fermat's last theorem.

- INTRODUCTORY GALOIS THEORY [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics.

- GROUP COHOMOLOGY [Alg2]
  Group cohomology is a basic and enormously important mathematical theory with applications in algebra, topology, and number theory. The project will study the initial theory staring with cohomology of discrete groups and then perhaps move on to cohomology of profinite groups. This project can be used as a platform for continuing with study of Galois cohomology and Selmer groups.

- THE THEOREM OF BILLING–MAHLER [Alg2, EllKurv]
  A big theorem of Barry Mazur (1977) implies in particular that if n is the order of a rational point of finite order on an elliptic curve defined over $\mathbb{Q}$ then either $1 \leq n \leq 10$ or $n = 12$. Thus, in particular, $n = 11$ is impossible. This latter statement is the theorem of Billing and Mahler (1940). The project studies the proof of the theorem of Billing–Mahler which will involve a bit more theory of elliptic curves as well as an initial study of algebraic number theory. The impossibility of $n = 13$ can also be proved with these methods.

- TORSION POINTS ON ELLIPTIC CURVES [Alg2, EllKurv]
  The project continues the study of elliptic curves defined over $\mathbb{Q}$ in the direction of a deeper study of (rational) torsion points. There are several possibilities here, for instance, parametrizations of curves with a point of a given, low order, generalizations of the Nagell–Lutz theorem, the structure of the group of torsion points on elliptic curves defined over a p-adic field (Lutz' theorem).

- PRIMALITY TESTING [Alg2]
  How can one decide efficiently whether a large number is a prime number? The project will study one or more of the mathematically sophisticated methods of doing this: the Miller–Rabin probabilistic primality test and/or the more recent Agrawal-Kayak-Saxena deterministic primality test. The project will include an initial study of algorithmic complexity theory.

- FACTORIZATION ALGORITHMS [Alg2]
  How can one find the prime factorization of a large number? The project will study one or more of the mathematically sophisticated methods of doing this: the Dixon factorization method, factorization via continued fractions, the quadratic sieve. The project will include an initial study of algorithmic complexity theory.

- OPEN PROJECT [?]
  If you have some ideas on your own for a project within the general area of number theory, you can always come and discuss the possibilities with me.

Previous projects:

- The Agrawal-Kayak-Saxena primality test [Alg2]
- Selmer groups and Mordell's theorem [Alg3, EllKurv]
- Hasse–Minkowski's theorem on rational quadratic forms [Alg2]
- Torsion points on elliptic curves [Alg2, EllKurv]
- Factorization via continued fractions [Alg2, Krypto]
- The Pohlig-Hellman algorithm for computing discrete logarithms [Alg2]
- Schoof's algorithm [Alg3, EllKurv]

3.3 Jørn B. Olsson

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Relevant interests:

Finite groups and their characters, finite symmetric groups and related topics from combinatorics and number theory

Suggested projects:

- Results on Permutation groups [Alg 2]
  Give a thorough description of selected abstract results on permutation groups, supplemented by concrete explicit examples.
  Literature: H. Kurzweil-B. Stellmacher, Theory of finite groups / D. Passman, Permutation groups
- Some properties of finite solvable groups [Alg 2]
  There is a number of interesting results on finite solvable groups, which helps you understand some of their characteristic properties, for instance a generalization of Sylow's theorem. The purpose of the project is to present some of these results.
  Literature: D.J.S. Robinson, A Course in the Theory of Groups / M. Hall, Theory of Groups
- Some finite p-groups [Alg 2]
  A p-group is a group of prime power order. Such groups have a rich structure and there are many of them. Present some basic results and a number of concrete examples.
- Equations in finite groups [Alg 2]
  We consider equations on the form $x^n = c$, where c is an element of a finite group G. Give a proof of Frobenius' theorem on the number of solutions to such an equation and study more explicitly the case, where G is a symmetric group.

- GENERATORS AND RELATIONS IN GROUPS [Alg 2]
  Give a description of a free group and explain how a group may be defined by generators and relations on the generators. This should be illustrated by concrete examples.
  Literature: D.J.S. Robinson, A Course in the Theory of Groups / M. Hall, Theory of Groups

- GROUPS OF SMALL ORDER [Alg 2, Alg 3]
  Present some basic tools to study groups of a given finite order and apply them to "classify" groups of relatively small order.

- INTEGER PARTITIONS []
  There is a very extensive literature on integer partitions. They play a role in representation theory, in combinatorics and in number theory. Present some examples of simple basic results on partitions, based primarily on the book by Andrews and Eriksson and illustrate the results by examples.
  Literature: G.E Andrews - K. Eriksson, Integer Partitions

- THE ROBINSON-SCHENSTED CORRESPONDENCE AND ITS PROPERTIES [Alg 2]
  The Robinson-Schensted correspondence is an interesting natural bijection between the set of permutations and the set of pairs of so-called standard tableaux of the same shape, which is fairly easy to describe. Present the the definition of a standard tableau and of the Robinson-Schensted correspondence and illustrate some of its basic properties.
  Literature: B. Sagan, The Symmetric Group

- STANDARD TABLEAUX AND THE HOOK FORMULA [Alg 2]
  This project is of a combinatorial nature and of relevance for the representation theory of symmetric groups. The surprisingly nice hook formula tells you what the number of standard tableux of given shape is. Present the definitions of partitions, of hooks in partitions, of standard tableaux of a given (partition) shape and prove the brancing rule for standard tableaux and then the hook formula, using an inductive argument. Illustrate with explicit examples.

- SPECHT MODULES FOR SYMMETRIC GROUPS [Alg 2]
  This is a basic construction in representation theory of symmetric groups. Give a brief introduction to the group algebra and its modules and describe the irreducible modules in the case of the symmetric groups.
  Literature: B. Sagan, The Symmetric Group

3.4 Other projects

Other projects in this area can be found with

- Tarje Bargheer(7.2)
- Christian Berg (4.1)

- Alexander Berglund (7.3)
- Jesper Grodal (7.3)
- Morten S. Risager (4.5)

4 Analysis

4.1 Christian Berg

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Relevant interests:

Orthogonal polynomials and moment problems. Complex analysis. Commutative harmonic analysis.

Suggested projects:

- THE GAMMA FUNCTION [An1,KomAn]
  Euler's Gamma function is the most import of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.

- ENTIRE FUNCTIONS [An1, Koman]
  Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.

- FIBONACCI NUMBERS [An1]
  The Fibonacci numbers 0,1,1,2,3,5,... are determined by taking the sum of the previous two numbers to get the next. They occur in many different areas of mathematics and have interesting number theoretical properties. Furthermore they have connections to the theory of orthogonal polynomials, cf. `www.math.ku.dk/~berg/manus/normathilbert.pdf`.

Previous projects:

- SPHERICAL FUNCTIONS [An1]

- CONFORMAL MAPPING [An1, KomAn]

- TOPOLOGICAL GROUPS, HAAR MEASURE [An1,MI]

4.2 Bergfinnur Durhuus

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Relevant interests:

Analysis: Operator theory, differential equations. Mathematical physics: Quantum mechanics, statistical mechanics. Discrete mathematics: Graph theory, analytic combinatorics, complexity theory,

Suggested projects:

- GRAPH COLOURING PROBLEMS [Dis1, An1]
  Problems originating from various areas of mathematics can frequently be formulated as colouring problems for certain types of graphs. The four-colour problem is probably the best known of coulouring problems but there is a variety of other interesting colouring problems to attack

- COMBINATORICS OF GRAPHS [Dis1, An1, ComAn]
  Counting of graphs specified by certain properties (e.g. trees) is one of the classical combinatorial problems in graph theory having applications in e.g. complexity theory. The method of generating functions is a particularly effective method for a large class of such problems making use of basic results from complex analysis

- UNBOUNDED OPREATORS AND SELF-ADJOINTNESS [An2]
  Many of the interesting operators playing a role in mathematical physics, in particular differential operators of use in classical and quantum mechanics, are unbounded. The extension of fundamental results valid for bounded operators on a Hilbert space, such as the notion of adjoint operator and diagonalisation properties, is therefore of importance and turns out to be non-trivial

Previous projects:

- CLIFFORD ALGEBRAS, SPIN GROUPS AND DIRAC OPERATORS [Alg1,An2]
- RAMSEY THEORY [Dis1,An1]
- CAUSAL STRUCTURES [An1,Geom2]
- THE TUTTE POLYNOMIAL [Dis1,An1]
- KNOT THEORY AND STATISTICAL MECHANICS [Dis1,AN1]
- GRAPH 3-COLOURINGS [Dis1,An1]
- MINIMAL SURFACES [Geom1,An1]
- PLANAR GRAPHS [Dis1,AN1]

4.3 Jens Hugger

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Relevant interests:

Numerical analysis – eScience

Suggested projects:

- Convergence of numerical methods for PDE's [An2]
Learn the theory of convergence analysis fro numerical methods for PDE's. Apply the theory to a real life problem (of your choice or provided by me like for example the Asian option from finance theory.

- Numerical methods for differential equations [NumIntro, NumDiff]
Pick a differential equation and solve it with a numerical method. Either bring your own problem or get one from the advisor.

- Numerical methods for interpolation or integration in several dimensions or iterative solution of large equation systems [NumIntro]
Pick a problem and solve it with a numerical method. Either bring your own problem or get one from the advisor.

- Porting part of a Maple program into a fast programming language [NumIntro, Computer science en masse]
Replace the slow part of a Maple code for solving an Asian option with code written in a faster language. Write help pages or manuals about how to do this, to be used in a bachelor level course.

Previous projects:

- Convection-diffusion in one variable [NumDiff]

- Asian options [NumDiff]

4.4 Enno Lenzmann

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Relevant interests:

Analysis, Partial Differential Equations, Mathematical Physics.

Suggested projects:

- The Wave Equation [An1, An2]
The wave equation is a fundamental partial differential equation in physics (e. g., propagation of waves and relativistic quantum mechanics). In this project, you are supposed to learn and develop the basic rigorous theory for the (linear) wave equation, followed by some peeks into the nonlinear wave equation.

- Nonlinear Schrödinger Equations [An1, An2]
Nonlinear Schrödinger equations describe interesting physical phenomena ranging from nonlinear optics to ultra-cold atoms (Bose-Einstein condensation).

Here you will study basic mathematical results about the nonlinear Schrödinger equation, with an emphasize on so-called soliton solutions.

4.5 Morten S. Risager

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Relevant interests:

Number theory, automorphic forms, complex analysis, Riemann surfaces.

Suggested projects:

- THE PRIME NUMBER THEOREM [KomAn, An2]
  The prime number theorem gives a quantitative version of Euclid theorem about the infinitude of primes: it describes how the primes are distributed among the integers. It was conjectured 100 years before the first proof.
- TWIN PRIMES AND SIEVE THEOREMS [KomAn, An2]
  Very little is known about the number of twin primes. Using sieve methods one can show that the sum of reciprocicals of twin primes is convergent. Still it is not known if there are only finitely many or not.
- THE FUNCTIONAL EQUATION FOR RIEMANN'S ZETA FUNCTION [KomAn, An2]
  Using methods from Fourier analysis - in particular Poisson summation - one investigates the properties of Riemann's famous zeta function.
- COUNTING ELEMENTS IN FREE GROUPS [KomAn, An2]
  How does one count in a resonable way the number of elements in the free group on n generators? Using methods from linear algebra one can give good asymptotic and statistical results. Numerical investigations is also a possibility.

Previous projects:

- ELEMENTARY METHODS IN NUMBER THEORY, AND A THEOREM OF TERRENCE TAO. [An2, ElmTal]
- PRIMES IN ARITHMETIC PROGRESSIONS [KomAn, An2]
- SMALL EIGENVALUES OF THE AUTOMORPHIC LAPLACIAN AND RADEMACHERS CONJECTURE FOR CONGRUENCE GROUPS [KomAn, An3]

4.6 Other projects

Other projects in this area can be found with

- Thomas Danielsen (5.1)

5 Geometry

5.1 Thomas Danielsen

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Relevant interests:

Representation theory for Lie algebras. Mathematical physics. Geometric analysis.

Suggested projects:

- Representation Theory for Lie Algebras [LinAlg]
  Introduction to Lie algebras and their representations. The aim is to classify the irreducible representations of the Lie algebras $\mathfrak{sl}(2,\mathbb{C})$ and $\mathfrak{sl}(2,\mathbb{C})$, possibly with applications to quantum mechanics (the harmonic oscillator and angular momentum)
- Fourier Theory on Abelian Groups [MI, An3, Top, knowledge of Banach algebras]
  Inspired by the classical theory of the Fourier transform on $\mathbb{R}^n$, the aim of this project is to define a Fourier transform on locally compact abelian groups and to develop a theory similar to the $\mathbb{R}^n$ case, i.e. inversion formula and Plancherel formula.
- Gauge Field Theory [Geom2]
  Gauge theory is the physical theory used to describe interaction between particles, such as electromagnetic and weak and strong nuclear forces. In this project the aim is to give an elegant formulation of gauge theory in terms of so-called principal bundles and connections. No prior knowledge of physics is required.

5.2 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- Global properties of curves (and/or surfaces) [Geom1,An1]
  The differential geometry studied in Geometry 1 is of a local nature. The curvature of a curve in a point, for example, describes a property of the curve just in the vicinity of that point. In this project the focus is on *global* aspects of closed curves, as for example expressed in Fenchel's theorem, which gives a lower bound for the total integral of the curvature, in terms of the perimeter.

- Geodesic distance [Geom1,An1]
  The geodesic distance between two points on a surface is the shortest length of a geodesic joining them. It turns the surface into a metric space. The project consists of describing some propreties of the metric. For example Bonnet's theorem: *If the Gaussian curvature is everywhere* ≥ 1, *then all distances are* $\leq \pi$.

- The Heisenberg group [An1,An2]
  The Heisenberg group is important, for example because it is generated by the position and momentum operators in quantum mechanics. The purpose of this project is to study its representation theory. A famous theorem of Stone and von Neumann relates all irreducible representations to the Schrödinger representation acting on $L^2(\mathbf{R}^n)$.

- Uncertainty principles [An1,Sand1,KomAn]
  Various mathematical formulations of the Heisenberg uncertainty principle are studied. Expressed mathematically, the principle asserts that a non-zero function f on $\mathbf{R}$ and its Fourier transform $\hat{f}$ cannot be simultaneously concentrated. A precise version, called the Heisenberg inequality, expresses this in terms of standard deviations. A variant of the theorem, due to Hardy, states that f and $\hat{f}$ cannot both decay more rapidly than a Gaussian function.

- The Peter-Weyl theorem [An1,An2,Sand1]
  The purpose of this project is to study $L^2(G)$ for a compact group G, equipped with Haar measure. The theorem of Peter and Weyl describes how this space can be orthogonally decomposed into finite dimensional subspaces, which are invariant under left and right displacements by G. Existence of Haar measure can be proved or assumed.

5.3 Other projects

Other projects in this area can be found with

- Ib Madsen (7.5)
- Nathalie Wahl (7.7)

6 Noncommutativity

6.1 Erik Christensen

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Relevant interests:

Group Algebras, Non Commutative Geometry, Fractal Sets, Convexity, Operator Algebras.

Suggested projects:

- DISCRETE GROUPS AND THEIR OPERATOR ALGEBRAS [Analysis 3]
  Many aspects of discrete groups are reflected in the operator algebras generated by their unitary representations.
- ELEMENTARY ASPECTS OF NON COMMUTATIVE GEOMETRY APPLIED TO FRACTAL SETS [Analysis 3]
  Even though fractal sets are quite far from being smooth, it is possible to describe parts of the geometry of a Cantor set or the Sierpinski Gasket using tools from non commutative geometry
- CONVEXITY AND DISCRETE GEOMETRY [Analysis 1]
  Convex sets have many nice properties and the methods used fit quite naturally with familiar arguments from the plane or the 3-dimensional space. There is a lot of difficult problems which may be reached even for a bachelor student.
- EXERCISES ON OPERATOR ALGEBRA [Analysis 3]
  Based on the course Analysis 3 you may want to learn more on certain aspects of operator algebras. This project consists in reading a text and demonstrating your understanding of the items read by solving several exercises.

6.2 Henrik Densing Petersen

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Relevant interests:

Operator algebras, geometric / measurable group theory, invariant percolation, universal coding in information theory.

For a more thorough project description please see:
http://www.math.ku.dk/~m03hdp/hdp_bpkatalog.pdf

Suggested projects:

- GROUP ACTIONS AND MEASURABLE EQUIVALENCE RELATIONS [MI, Alg1 (or some basic knowledge of group theory)]

One currently very popular way to study countable groups is through their actions on the unit interval with Lebesgue measure. The basic goal of this project is to study how much the equivalence relations induced by such actions tell us about the groups themselves, to construct examples of explicit actions and prove basic results about these.

This project can be taken in several directions, and the scope and prerequisites can be adjusted to fit the individual student.

- PROPERTY (T) FOR (DISCRETE) GROUPS [An3(possibly just An2), (Top), MI, Alg1, some knowledge of representation theory is a plus but can also be included in the project]
  Property (T) is a rigidity property for infinite groups introduced by Kazhdan in 1967 to prove for instance that $SL_3(\mathbb{Z})$ is finitely generated. Recall that this is the group of 3×3 matrices with integer entries and determinant 1. Property (T) for a group G is usually defined in terms of its representation theory, and studying this then allows one to deduce, often very strong, results about G.

- UNIVERSAL CODING IN INFORMATION THEORY [MI, An1, some knowledge of programming (possibly in Maple) is a big plus]
  The first goal of this project is to attain an understanding and working knowledge of previously established results on universal coding.

  The second goal is, using computer experiments, to calculate (approximately) universal codes for "alot" of examples outside of the previously understood cases and to see if we can put forward some more general conjectures concerning other classes.

6.3 Søren Eilers

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Relevant interests:

Advanced linear algebra related to operator algebras. Dynamical systems. Mathematics in computer science; computer science in mathematics.

Suggested projects:

- PERRON-FROBENIUS THEORY WITH APPLICATIONS [LinAlg, An1]
  Methods involving matrix algebra lead to applications such as Google's PageRank and to the ranking of American football teams.

- DATA STORAGE WITH SYMBOLIC DYNAMICS [An1, Dis1]
  Engineering constraints neccessitate a recoding of arbitary binary sequences into sequences meeting certain constraints such as "between two consecutive ones are at least 1, and at most 3, zeroes". Understanding how this is done requires a combination of analysis and discrete mathematics involving notions such as entropy and encoder graphs.

- Experimental mathematics [LinAlg]
  Design an experiment in Maple to investigate a mathematical problem, cf. `www.math.ku.dk/~eilers/xm`.

Previous projects:

- An experimental approach to flow equivalence [An1]
- Visualization of non-euclidean geometry [MatM, Geom1]
- Planar geometry in high school mathematics [MatM]
- Liapounov's theorem [MI]

6.4 Niels Grønbæk

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Relevant interests:

Banachrum, banachalgebra, kohomologi, matematikkens didaktik

Suggested projects:

- Et undervisningsforløb på gymnasialt niveau [LinAlg, An1, Alg1, Geo1]
  Projektet går ud på at tilrettelægge, udføre og evaluere et undervisningsforløb af ca. 2 ugers varighed i en gymnasieklasse.

Suggested projects:

- Amenable Banach Algebras [An3]
  Amenability of Banach algebras is an important concept which originates in harmonic analysis of locally compact groups. In the project you will establish this connection and apply it to specific Banach algebras such as the Banach algebra of compact operators on a Hilbert space.

6.5 Magdalena Musat

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Relevant interests:

Banach Spaces, Functional Analysis, Operator Algebras, Probability Theory

Suggested projects:

- Geometry of Banach spaces [Analysis 3]
  A number of very interesting problems concerning the geometry of Banach spaces can be addressed in a bachelor project. For example, does every infinite dimensional Banach space contain an infinite dimensional reflexive subspace or an isomorphic copy of l_1 or c_0? Or, does there exist a reflexive Banach space in which neither an l_p-space, nor a c_0-space can embed? Another project could explore the theory of type and cotype, which provides a scale for measuring how close a given Banach space is to being a Hilbert space.

- Convexity in Banach spaces [Analysis 3]
  The question of differentiability of the norm of a given Banach space is closely related to certain convexity properties of it, such as uniform convexity, smoothness and uniform smoothness. This project will explore these connections, and study further properties of uniformly convex (respectively, uniformly smooth) spaces. The Lebesgue spaces L_p $(1 < p < \infty)$ are both uniformly convex and uniformly smooth.

- Haar measure [MI]
  This project is devoted to the proof of existence and uniqueness of left (respectively, right) Haar measure on a locally compact topological group G. For example, Lebesgue measure is a (left and right) Haar measure on $\mathbb{R}$, and counting measure is a (left and right) Haar measure on the integers (or any group with the discrete topology).

- Fernique's theorem [SAND 1, Analysis 3]
  This project deals with probability theory concepts in the setting of Banach spaces, that is, random variables taking values in a (possibly infinite dimensional) Banach space. Fernique's theorem generalizes the result that gaussian distributions on $\mathbb{R}$ have exponential tails to the (infinite dimensional) setting of gaussian measures on arbitrary Banach spaces.

6.6 Ryszard Nest

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Relevant interests:

Non-Commutative Geometry, Deformation Theory, Poisson Geometry

Suggested projects:

- Clifford Algebras [LinAlg, Geom 1]
  Clifford algebra is a family $\mathcal{C}^{p,q}$ of finite dimensional algebras associated to non-degenerate bilinear forms which play very important role in both topology and geometry. The simplest examples are $\mathbb{R}$, $\mathbb{C}$ and the quaternion algebra $\mathbb{H}$. The main result is the periodicity modulo eight of $\mathcal{C}^{p,q}$, which has far reaching

consequences (e.g., Bott periodicity, construction of Dirac operators) in various areas of mathematics.

- Axiom of choice and the Banach-Tarski paradox [LinAlg, Analysis 1]
  The axiom of choice, stating that for every set of mutually disjoint nonempty sets there exists a set that has exactly one member common with each of these sets, is one of the more "obvious" assumptions of set theory, but has far reaching consequences. Most of modern mathematics is based on its more or less tacit assumption. The goal of this project is to study equivalent formulations of the axiom of choice and some of its more exotic consequences, like the *Banach-Tarski paradox*, which says that one can decompose a solid ball of radius one into five pieces, and then rearrange those into two solid balls, both with radius one.

- Formal deformations of $\mathbb{R}^{2n}$ [LinAlg, Geom 1]
  The uncertainty principle in quantum mechanics says that the coordinate and momentum variables satisfy the relation $[p, x] = \hbar$, where $\hbar$ is the Planck constant. This particular project is about constructing associative products in $C^\infty(\mathbb{R}^{2n})[[\hbar]]$ satisfying this relation and studying their properties.

6.7 Otgonbayar Uuye

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Relevant interests:

Non-Commutative Geometry

Suggested projects:

- Banach-Tarski Paradox [An1]
  In 1924, S. Banach and A. Tarski showed that one can divide a ball into finitely many pieces and reassemble them to get two balls identical to the original one. But how is that possible? We have a paradox! Or not. This apparent paradox can be explained using the *non-amenability* of the motion group and the *axiom of choice*.

- Compact Groups and the Peter-Weyl Theorem [An2, LinAlg]
  Compact groups and their representations appear in many fields of mathematics and physics. The Peter-Weyl theorem is the fundamental result that governs the representation theory of compact groups. There are many proofs known. Modern proofs use the spectral theory of self-adjoint compact operators on a Hilbert space. For matrix groups, the Stone-Weierstrass theorem suffices.

6.8 Mikael Rørdam

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Relevant interests:

Operator Algebras, Topics in Measure Theory, Discrete Mathematics

Suggested projects:

- Topics in C^*-algebras [Analysis 3]
  C^*-algebras can be defined either abstractly, as a Banach algebra with an involution, or concretely, as subalgebras of the algebra of bounded operators on a Hilbert space. They can be viewed as non-commutative analogues of spaces, since every commutative C^*-algebra is equal to the set of continuous functions on a locally compact Hausdorff space. Several topics concerning C^*-algebras and concerning the study of specific examples of C^*-algebras, can serve as interesting topics for a bachelor project.

- Topics in measure theory [MI]
  We can here look at more advanced topics from measure theory, that are not covered in MI, such as existence (and uniqueness) of Lebesgue measure, or more generally of Haar measure on locally compact groups. Results on non-measurability are intriguing, perhaps most spectacularly seen in the Banach-Tarski paradox that gives a recipe for making two solid balls of radius one out of a single solid ball of radius one!

- Topics in discrete mathematics [Dis2 & Graf]
  One can for example study theorems about coloring of graphs. One can even combine graph theory and functional analysis and study C^*-algebras arising from graphs and the interplay between the two (in which case more prerequisites are needed).

Previous projects:

- Irrational and rational rotation C^*-algebras [Analyse 3]
- Convexity in functional analysis [Analyse 3]
- The Banach-Tarski Paradox [MI recommended]

7 Topology

7.1 David Ayala

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Relevant interests:

Algebraic topology and its relationship to locally defined spaces such as manifolds.

Suggested projects:

- Vector fields and Euler characteristic [basic calculus, linear algebra, point-set topology]
  Showing why it is impossible to comb a hairy sphere. More specifically, describing a relationship between a global invariant and a local one.
- Curves and surfaces in 3-space [vector calculus]
  Understanding curves and surfaces in $\mathbb{R}^3$. Defining a notion of curvature and torque of a curve, and defining a notion of curvature of a surface. Mathematically describing surfaces which minimize area such as soap bubbles.
- The Hopf map and its relevance [basic point-set topology]
  Finding a non-trivial map from the boundary of a 4-dimensional ball to the boundary of a 3-dimensional ball. There are beautiful pictures. The ideas can be generalized in many directions depending on interest; for instance: projective spaces, bundles, quaternions, two-dimensional orbifolds,...(you don't need to know what these words mean).
- The sphere eversion [vector calculus and basic point-set topology]
  To explain how it is possible to turn a sphere inside out without pinching it. There are beautiful pictures. It is a good way to see the relationship between geometric ideas and topological ideas.

7.2 Tarje Bargheer

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Relevant interests:

Geometric objects; manifolds, knots and string topology – and algebraic structures hereon.

Suggested projects:

- Khovanov Homology [AlgTop – or familiarity with category theory]
  The complexity of knots is immense. Explore `http://katlas.org/`. Over the

last 100 years various tools have been developed to distinguish and classify knots. A lot of work is still needed to have a good understanding of the world of knots.

This project would aim at understanding one of the stronger tools available to this date; Khovanov Homology.

- Operads and Algebras [Alg2]
  Operads is an effective tool to cope with exotic algebraic structures. How do you for instance work with algebraic structures that are not (strictly) associative? A framework for given a broader perspective on various types of algebras would be developped. Depending on interest, pointers towards geometric and topological algebraic aspects is also a possibility.

  Nathalie Wahl is also a potential supervisor on this project.

- Morse Theory [Geom2 – for instance simultaneously]
  The 2. derivative test, known from MatIntro, tells you about local characteristica of a 2-variable function. Expanding this test to manifolds in general yields Morse Theory, which plays a key role in modern geometry.

  This project would start out by introducing Morse Theory. Various structure and classification results about manifolds could be shown as applications of the theory.

7.3 Alexander Berglund

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Relevant interests:

Algebra, combinatorics, topology.

Suggested projects:

- Topological combinatorics [Dis1, Top]
  Combinatorial problems, such as determining chromatic numbers of graphs, can be solved using topological methods.

- Partially ordered sets [Dis1]
  Partially ordered sets are fundamental mathematical structures that lie behind phenomena such as the Principle of Inclusion-Exclusion and the Möbius inversion formula.

- Simplicial complexes in algebra and topology [Alg1, Top]
  The goal of this project is to understand how simplicial complexes can be used to set up a mirror between notions in topology and algebra. For instance, the algebraic mirror image of a topological sphere is a Gorenstein ring.

7.4 Jesper Grodal

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Relevant interests:

Topology, Algebra, Geometry.

Suggested projects:

- Group cohomology [Alg2]
  To a group G we can associate a collection of abelian groups $H^n(G)$, $n \in \mathbf{N}$, containing structural information about the group we started with. The aim of the project would be to define these groups, examine some of their properties, and/or examine applications to algebra, topology, or number theory. See e.g.: K.S. Brown: Cohomology of groups

- Group actions [Top, Alg2]
  How can groups act on different combinatorial or geometric objects? Eg. which groups can act freely on a tree? See e.g.: J.-P. Serre: Trees.

- The Burnside ring [Alg2]
  Given a group G we can consider the set of isomorphism classes of finite G-sets. These can be "added" and "multiplied" via disjoint union and cartesian projects. By formally introducing additive inverses we get a ring called the Burnside ring. What's the structure of this ring and what does it have to do with the group we started with? See:
  `http://en.wikipedia.org/wiki/Burnside_ring`

- The classification of finite simple groups [Alg2]
  One of the most celebrated theorems in 20th century mathematics gives a complete catalogue of finite simple groups. They either belong to one of three infinite families (cyclic, alternating, or classical) or are one of 26 sporadic cases. The aim of the project is to explore this theorem and perhaps one or more of the sporadic simple groups. See:
  `http://en.wikipedia.org/wiki/Classification_of_finite_simple_groups`

- The Platonic solids and their symmetries [Top, Alg2]
  A Platonic solid is a convex polyhedron whose faces are congruent regular polygons, with the same number of faces meeting each vertex. The ancient greeks already knew that there were only 5 platonic solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The aim of the project is to understand the mathematics behind this. See: `http://en.wikipedia.org/wiki/Platonic_solid`

- Topological spaces from categories [Top, Alg2]
  Various algebraic or combinatorial structures can be encoded via geometric objects. These "classifying spaces" can then be studied via geometric methods. The goal of the project would be to study one of the many instances of these this, and the project can be tilted in either topological, categorical, or combinatorial directions. See e.g.: A. Björner, Topological methods. Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.

Previous projects:

- Steenrod operations—construction and applications [AlgTopII]
- Homotopy theory of topological spaces and simplicial sets [AlgTopII]

7.5 Ib Madsen

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Relevant interests:

Homotopy theory, topology of manifolds.

Suggested projects:

- de Rham cohomology []
- Poincaré duality []
- Covering spaces and Galois Theory []
- The Hopf invariant []

7.6 Jesper Michael Møller

moller@math.ku.dk

Relevant interests:

All kinds of mathematics.

Suggested projects:

- Poincaré sphere [Topology, group theory]
  What are the properties of the Poincaré sphere?

Suggested projects:

- Chaos [General topology]
  What is chaos and where does it occur?

Suggested projects:

- PROJECT OF THE DAY [Mathematics]
  http://www.math.ku.dk/~moller/undervisning/fagprojekter.html

7.7 Nathalie Wahl

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Relevant interests:

Graphs, surfaces, 3-dimensional manifolds, knots, algebraic structures.

Suggested projects:

- KNOTS [Alg1,Top]
  Mathematically, knots are embeddings of circles in 3-dimensional space. They are rather complicated objects that can be studied combinatorially or via 3-manifolds. The project consists of learning some basics in knot theory. See for example http://www.earlham.edu/~peters/knotlink.htm.

- BRAID GROUPS, CONFIGURATION SPACES AND LINKS [Alg1,Top]
  The braid group on n strands can be defined in terms of braids (or strings), or as the fundamental group of the space of configurations of n points in the plane. It is related to knots and links, and also to surfaces. The project consists of exploring braid groups or related groups like mapping class groups. See for example J. Birman, Braids, links, and mapping class groups.

- CLASSIFICATION OF SURFACES [Top,Geom1]
  Closed 2-dimensional surfaces can be completely classified by their genus (number of holes). There are several ways of proving this fact and the project is to study one of the proofs. See for example W. Massey, A Basic Course in Algebraic Topology, or A. Gramain, Topology of Surfaces.

- 3-MANIFOLDS [Top,Geom1]
  3-dimensional manifolds are a lot harder to study than 2-dimensional ones. The geometrization conjecture (probably proved recently by Perelman) gives a description of the basic building blocks of 3-manifolds. Other approaches to 3-manifolds include knots, or "heegaard splittings", named after the Danish mathematician Poul Heegaard. The project consists of exploring the world of 3-manifolds. See for example http://en.wikipedia.org/wiki/3-manifolds.

- NON-EUCLIDEAN GEOMETRIES [Geom1]
  Euclidean geometry is the geometry we are used to, where parallel lines exist and never meet, where the sum of the angles in a triangle is always 180°. But there are geometries where these facts are no longer true. Important examples are the hyperbolic and the spherical geometries. The project consists of exploring non-euclidian geometries. See for example
  http://en.wikipedia.org/wiki/Non-euclidean_geometries

- FROBENIUS ALGEBRAS, HOPF ALGEBRAS [LinAlg,Alg1]
  A Frobenius algebra is an algebra with extra structure that can be described algebraically or using surfaces. A Hopf algebra is a similar structure. Both types of algebraic structures occur many places in mathematics. The project consists of looking at examples and properties of these algebraic structures. See for example J. Kock, Frobenius algebras and 2D topological quantum field theories.

8 History and philosophy of mathematics

8.1 Jesper Lützen

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Relevant interests:

History of Mathematics

Suggested projects:

- THE HISTORY OF NON-EUCLIDEAN GEOMETRY [Hist1, preferrably VtMat]
  How did non-Euclidean geometry arise and how was its consistency "proved". How did the new geometry affect the epistemology of mathematics?
- THE DEVELOPMENT OF THE FUNCTION CONCEPT [Hist1]
  How did the concept of function become the central one in mathematical analysis and how did the meaning of the term change over time.
- ARCHIMEDES AND HIS MATHEMATICS [Hist1]
  Give a critical account of the exciting life of this first rate mathematician and analyze his "indivisible" method and his use of the exhaustion method.
- WHAT IS A MATHEMATICAL PROOF, AND WHAT IS ITS PURPOSE [Hist1, VtMat]
  Give philosophical and historical accounts of the role(s) played by proofs in the development of mathematics

Previous projects:

- A BRIEF HISTORY OF COMPLEX NUMBERS [Hist1, preferrably KomAn]
- MATHEMATICAL INDUCTION. A HISTORY [Hist1]
- ASPECTS OF EULER'S NUMBER THEORY [Hist1, ElmTal]
- MATHEMATICS IN PLATO'S DIALOGUES [Hist1, VtMat]
- AXIOMATIZATION OF GEOMETRY FROM EUCLID TO HILBERT [Hist1, preferrably VtMat]
- LAKATOS' PHILOSOPHY APPLIED TO THE FOUR COLOR THEOREM [Dis, Hist1]
- HISTORY OF MATHEMATICS IN MATHEMATICS TEACHING: HOW AND WHY [Hist1, DidG preferrably DidMat]

9 Other areas

9.1 Discrete mathematics

Projects in this area can be found with

- Alexander Berglund (7.3)
- Bergfinnur Durhuus (4.2)
- Søren Eilers (6.3)
- Jørn B. Olsson (3.3)
- Mikael Rørdam (6.8)

9.2 Teaching and didactics in mathematics

Projects in this area can be found with

- Niels Grønbæk (6.4)
- Jesper Lützen (8.1)

9.3 Aspects of computer science

Projects in this area can be found with

- Søren Eilers (6.3)
- Jens Hugger (4.3)

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