

## 2-dimensional systems of ordinary differential equations

The following exercises must be handed in in a hand-written format. *Deadline:* Tuesday, 30th of October, 4pm.

1. (4 points). Consider the system of ordinary differential equations

$$\begin{aligned}x' &= -2x + y, \\y' &= 4x - 5y.\end{aligned}$$

Show that there is only one equilibrium point, which is stable. Find the nullclines and sketch the picture of the phase space for the system of ordinary differential equations. Try to guess if the equilibrium point is globally stable.

2. (4 points). Consider the system of differential equations

$$\begin{aligned}x' &= v, \\v' &= -x,\end{aligned}$$

describing the movement of a spring.

- (i) Find the general expression of a trajectory and draw a picture of the phase space  $(x, v)$  with the trajectories, deciding in which direction is the movement.
- (ii) Show that there is only one equilibrium point. What is its stability? Justify your answer both in a numerical way and by looking at the phase space trajectories.
- (iii) Consider now the system describing the movement of the spring, but now taking into account some friction:

$$\begin{aligned}x' &= v, \\v' &= -x - bv,\end{aligned}$$

with  $b > 0$ . Determine the equilibrium points and their stability.