Matrices and systems of linear equations

The following exercises must be solved with Octave. You have to write the answer of the questions on the assignment sheet prepared on the course webpage. You also have to upload the Octave history file. The points will be given according to your answer, and not to the Octave history file.

Deadline: Tuesday, 2nd of October, 4pm.

Let $A = \begin{pmatrix} 1 & 2 & -2 \\ 4 & 5 & -1 \\ 3 & -7 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 6 & 2 \\ 1 & -9 & 5 \\ 1 & -3 & 7 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 4 & -2 \\ 0 & -6 & -1 \\ -2 & 2 & -5 \\ -3 & 6 & -7 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$.

1. *(0.5 points).* Compute the matrix product of A with B; compute the product element by element of the matrices A and B.

2. *(0.5 points).* Find the determinant of A and the determinant of B. Is A invertible? Is B invertible? If yes, find the inverse.

3. *(1 point).* Find the transpose of C, C^t. Determine the rank of C. Find a minor in C, of order equal to the rank of C, whose determinant is different from zero. Can we find a minor in C, with determinant different from zero, and of order greater than the rank of C? Can we compute the determinant of C?

4. *(0.5 points).* Which is the element indexed $(2, 3)$ in the matrix A? Which is the element indexed $(3, 2)$ in the matrix A?

5. *(0.5 points).* When we try to make the matrix product $A{\cdot}C$ with "Octave", we get an error. Why is that?

6. *(0.5 points).* Write the "Octave" code to get the matrix C with the number "2" subtracted from every entry in the diagonal of C.

7. *(0.5 points).* Write the "Octave" code to merge the matrix A and the column vector v in one matrix.

8. *(2 points).* Discuss the following systems of linear equations: without solving them, but with the help of "Octave", decide whether there exists no solution, a unique solution, of infinitely many solutions, based on the comparison of ranks. Find also the Gaussian elimination of the augmented matrix, and compare the results. Find the solutions for the systems with solution.

$$\text{(i)} \begin{cases} x - 3y + 2z = 6 \\ 2x + y - 5z = -4 \\ 2x - 13y + 13z = 28 \end{cases}, \qquad \text{(ii)} \begin{cases} 3x + 2y + z = 1 \\ 5x + 3y + 4z = 2 \\ x + y - z = 1 \end{cases},$$

$$\text{(iii)} \begin{cases} x - 3y + 2z = -6 \\ 2x + y + -5z = 4 \\ 4x - 5y - z = -8 \end{cases}, \qquad \text{(iv)} \begin{cases} 2x + 3y + z + 2t = 4 \\ 4x + 3y + z + t = 5 \\ 5x + 11y + 3z + 2t = 2 \\ 2x + 5y + z + t = 1 \\ x - 7y - z + 2t = 7 \end{cases},$$