Vector spaces

The following exercises and questions must be handled over right before the class of the 9th of October starts.

Part 1: Theoretical questions.

1. *(0.5 points).* If we are given 3 vectors in a vector space F of dimension 3, can we assure that they form a basis of F?

2. *(0.5 points).* Consider a matrix A of rank 4. What is the dimension of the space generated by the columns of the matrix A?

3. *(0.5 points).* Consider a square matrix A of order n whose determinant is zero. Then, is it true that the rank of the matrix is $n-1$?

4. *(0.5 points).* Consider 4 linearly independent vectors in a vector space of dimension 4. Is it true that they form a basis of the vector space?

5. *(0.5 points).* Consider a set of 4 vectors that generate a vector space of dimension 4. Is it true that they form a basis of the vector space?

Part 2: Hand-written exercises. The following exercises must be solved hand-written. All the computations must be detailed.

6. *(1 point).* Show that the vectors $\{(1,2),(0,3)\}$ form a basis of $\mathbb{R}^2$.

7. *(1.5 points).* Consider the vector subspace $F = \{(x,y,z) \in \mathbb{R}^3 |\ x - y = 0\}$.

(i) Find a basis of F.

(ii) What is the dimension of F?

(iii) Check that the vector $(1,1,4)$ belongs to F and find the coordinates of this vector in the basis you found in (i).

Part 3: Octave. *(1.5 points).* To be solved with Octave. The file must be uploaded in the assignment prepared for that.

8. Find the dimension of the vector space

$$F = \langle (1,3,4,0,-1),(2,3,7,-3,0),(3,6,11,-3,-1),(1,5,4,-2,0)\rangle \subset \mathbb{R}^5.$$

Give a basis of F.

9. Find the dimension of the vector space

$$F = \langle (-1,3,9,0),(2,1,-3,0),(3,6,11,0),(1,5,4,,0)\rangle \subset \mathbb{R}^4.$$

Give a basis of F.

10. Find the coordinates of the vector $(1,2,3,4)$ in the basis

$$\{(-1,0,1,0),(0,1,3,1),(2,1,-1,6),(1,-3,0,1)\}$$

of $\mathbb{R}^4$.