

Øvelse 2

1. Let $u(x)$ be a concave function. Show that $e^{-u(x)}$ is a convex function. Show, moreover, that if the set $\{x : u'(x) = 0\}$ consists of at most one point, then $e^{-u(x)}$ is strictly convex.

2. Consider a risk X such that $\mathbb{P}[X = L] = p = 1 - \mathbb{P}[X = 0]$ for some $p \in (0, 1)$. An agent carrying the risk X has utility function $u(y)$ and initial wealth w . The agent is offered the following insurance treaty. He can freely choose the insured sum $S \in [0, L]$, i.e. the insurer pays S if a claim occurs, and has to pay the premium $(1 + \lambda)pS$, where $\lambda \geq 0$.
 - a) Show that the full coverage $S = L$ is optimal if and only if $\lambda = 0$.
Hint: Take the derivative of $\mathbb{E}[u(w - \pi(S) - s(X))]$ with respect to S , where $s(X) = (X - S)^+$.
 - b) Find necessary and sufficient conditions for $S = 0$ to be optimal.
 - c) Assume now $u(y)$ is twice continuously differentiable and that $S = 0$ is not optimal. Denote the optimal sum to be insured by S_0 . Discuss how S_0 depends on w in the case of decreasing risk aversion and in the case of increasing risk aversion.
Hint: Differentiate $\mathbb{E}[u(w - \pi(S) - s(X))]$ with respect to S , equate to zero, yielding $S_0(w)$ implicitly. Next differentiate the obtained equation with respect to w and solve for the first derivative of $S_0(w)$. Do not forget to check that S_0 is a maximum.

3. Consider the following (utility) functions

Quadratic utility $u_1(y) = y - \frac{y^2}{2c}, \quad y \leq c, \quad c > 0.$

Exponential utility $u_2(y) = -e^{-cy}, \quad y \in \mathbb{R}, \quad c > 0.$

Logarithmic utility $u_3(y) = \log(c + y), \quad y > -c.$

HARA utility $u_4(y) = \frac{(y + c)^\alpha}{\alpha}, \quad y > -c, \quad 0 < \alpha < 1.$

HARA utility $u_5(y) = -\frac{(c - y)^\alpha}{\alpha}, \quad y < c, \quad \alpha > 1.$

- a) Show that the functions $u_k(y)$ are indeed utility functions.
 - b) Calculate the risk aversion functions of $u_k(y)$.
 - c) Which risk aversion functions are increasing, respectively decreasing?
4. Let X be a loss. Consider full insurance. Calculate the zero utility premium for
 - a) Quadratic utility $u(y) = y - \frac{y^2}{2c}, \quad y \leq c, \quad c > 0.$

- b) Exponential utility $u(y) = -e^{-cy}, \quad y \in \mathbb{R}, c > 0.$
- c) Logarithmic utility $u(y) = \log(c + y), \quad y > -c.$
- d) HARA utility $u(y) = \frac{(y + c)^\alpha}{\alpha}, \quad y \geq -c, 0 < \alpha < 1.$
- e) HARA utility $u(y) = -\frac{(c - y)^\alpha}{\alpha}, \quad y \leq c, \alpha > 1.$

Discuss in addition the dependence on the initial wealth.