A few study hints

1. **Cramér's theorem in one dimension.** What is the statistical motivation of the problem? How does large deviations "improve" over classical estimates? What is a precise statement of the large deviation principle and of Cramér's theorem, and how do you compute the rate function? What are the main ideas of the proof? What is the change of measure used in the proof? How do you find this change of measure? Does the theorem hold in general, or are there some classes of r.v.'s where the theorem is *not* useful?

2. **Cramér's theorem in higher dimensions: upper bound.** Precise statement of the theorem. What are the main ideas in the proof of the upper bound? What is the change of measure argument and how is it used here? Approximation by halfspaces; extensions to general sets. What are the assumptions needed for these aspects of the proof to work? What is a statement of the corresponding upper bound for Gärtner-Ellis sequences?

3. **Cramér's theorem in higher dimensions: lower bound.** Precise statement of the theorem. What are the main ideas in the proof of the lower bound? Change of measure argument. What are the assumptions needed in the proof? How can Cramér's theorem (upper and lower bound) be used to derive conditional limit theorems (cf. example from the lectures)? What is a statement of the corresponding lower bound for Gärtner-Ellis sequences?

4. **Multidimensional ruin problem.** Precise statement of the problem, and comparison with the classical one-dimensional problem. Precise statement of the main theorem. Computation of the rate function. Main ideas of the proof, especially of the upper bound. In particular, how is the change of measure *different* from Cramér's theorem? Approximation by halfspace. How do you compute the rate function? How can the multidimensional ruin problem be viewed (to some extent) as a special case of Mogulskii's theorem?

5. **Importance sampling.** General description of the problem: why do we need importance sampling? What is a criteria which tells us when importance sampling is efficient? Precise statement of results for importance sampling in the multidimensional ruin problem. Proof of the upper bound. Dominating points (unique minimizing point, etc.) and their role in importance sampling problems.

6. **Credit risk application.** Statement of the problem. Application of Cramér's theorem in the single-type case and the Gärtner-Ellis theorem in the multitype case. Computation of the cumulant generating function and rate function in the special case of this problem. Shifted measure and the Gibbs conditioning principle. Likely default distribution, etc., conditional on default.

7. **Portfolio theory application.** Statement of the general problem. Stutzer's large deviation approach: criteria for selecting an optimal portfolio. Application of large deviation results for this problem. Connections with utility functions.