Beyond classification of Cuntz-Krieger algebras

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Theorem (Restorff)

Filtrated $K$-theory $\overline{FK}$ stably classifies Cuntz-Krieger algebras satisfying condition (II).

Definition

Filtrated $K$-theory $\overline{FK}(A)$ of $A$ consist of all groups $K_*(J/I)$ with $I \subseteq J$ ideals in $A$, together with all maps

\[ K_*(J/I) \xrightarrow{i} K_*(K/I) \]

\[ K_*(K/J) \xleftarrow{r} K_*(J/I) \xleftarrow{\delta} K_*(K/J) \]

induced by $J/I \hookrightarrow K/I \twoheadrightarrow K/J$ when $I \trianglelefteq J \trianglelefteq K \trianglelefteq A$. 
• Does $\overline{FK}$ strongly classify the Cuntz-Krieger algebras up to stable isomorphism?
• Does $(\overline{FK}, [1_A])$ classify the Cuntz-Krieger algebras up to unital isomorphism?
• Does $\overline{FK}$ stably classify the purely infinite graph $C^*$-algebras with a finite ideal lattice?
• Let $A$ be a purely infinite, separable, nuclear $C^*$-algebra in the bootstrap class and with a finite ideal lattice. If $\overline{FK}(A) \cong \overline{FK}(B)$ for $B$ a Cuntz-Krieger algebra, is $A$ then stably isomorphic to a Cuntz-Krieger algebra?

**Theorem (Kirchberg)**

Let $A$ and $B$ be $O_\infty$-absorbing, separable, nuclear $C^*$-algebras with $\text{Prim}(A) \cong \text{Prim}(B) \cong X$. Then any $KK(X)$-equivalence between $A$ and $B$ lifts to an $X$-equivariant isomorphism between $A \otimes \mathbb{K}$ and $B \otimes \mathbb{K}$. 

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**Theorem (Meyer-Nest)**

For $X$ finite linear space, the UCT holds, i.e. the sequence

$$\text{Ext}(FK_X(A), FK_X(B)) \hookrightarrow KK_*(X; A, B) \twoheadrightarrow \text{Hom}(FK_X(A), FK_X(B))$$

is exact for all separable $C^*$-algebras $A$ and $B$ over $X$ where $A$ lies in the bootstrap class $B(X)$.

**Theorem (Bentmann)**

For $X$ finite accordion space, the UCT holds.

**Corollary**

For purely infinite, separable, nuclear $C^*$-algebras $A$ and $B$ with all simple subquotients in the bootstrap class and with $\text{Prim}(A) \cong \text{Prim}(B) \cong X$ finite accordion space, any isomorphism between $FK_X(A) = \overline{FK}(A)$ and $FK_X(B) = \overline{FK}(B)$ lifts to an $X$-equivariant isomorphism between $A \otimes K$ and $B \otimes K$. 
• Does $\text{FK}$ strongly classify the Cuntz-Krieger algebras up to stable isomorphism?
• Does $\text{FK}$ stably classify the purely infinite graph $C^*$-algebras with a finite ideal lattice?
• Let $A$ be a purely infinite, separable, nuclear $C^*$-algebra in the bootstrap class and with a finite ideal lattice. If $\text{FK}(A) \cong \text{FK}(B)$ for $B$ a Cuntz-Krieger algebra, is $A$ then stably isomorphic to a Cuntz-Krieger algebra?
Theorem (Meyer-Nest)

There exist a finite space $X_0$ and purely infinite, separable, nuclear $C^*$-algebras $A$ and $B$ with $\text{Prim}(A) \cong \text{Prim}(B) \cong X_0$ and $A, B \in \mathcal{B}(X_0)$ satisfying that

$$FK_{X_0}(A) \cong FK_{X_0}(B), \quad KK_*(X; A, B)^{-1} = \emptyset.$$
Theorem (Meyer-Nest)

For the space $X_0$ there exists a functor $FK'$ for which the sequence

$$\text{Ext}(FK'(A), FK'(B)) \hookrightarrow KK_*(X_0; A, B) \twoheadrightarrow \text{Hom}(FK'(A), FK'(B))$$

is exact for all separable C*-algebras $A$ and $B$ over $X_0$ where $A$ lies in the bootstrap class $B(X_0)$.

Theorem (A-Restorff-Ruiz)

For C*-algebras $A$ and $B$ over $X_0$ with $A$ of real rank zero, any isomorphism between $FK_{X_0}(A)$ and $FK_{X_0}(B)$ lifts to an isomorphism between $FK'(A)$ and $FK'(B)$.
Corollary

For purely infinite, separable, nuclear C*-algebras $A$ and $B$ with all simple subquotients in the bootstrap class, with $\text{Prim}(A) \cong \text{Prim}(B) \cong X_0$ and with $A$ of real rank zero, any isomorphism between $\text{FK}_{X_0}(A) = \overline{\text{FK}}(A)$ and $\text{FK}_{X_0}(B) = \overline{\text{FK}}(B)$ lifts to an $X_0$-equivariant isomorphism between $A \otimes K$ and $B \otimes K$. 