Corners of Cuntz-Krieger algebras

**Theorem (A-Ruiz)**

Let $E$ be a countable directed graph. TFAE:

- $C^*(E)$ is a Cuntz-Krieger algebra,
- $E$ is finite with no sinks,
- $C^*(E)$ is unital and $\text{rank } K_0(C^*(E)) = \text{rank } K_1(C^*(E))$.

**Theorem (A-Ruiz)**

Let $A$ be a unital $C^*$-algebra and assume that $A$ is stably isomorphic to a Cuntz-Krieger algebra. Then $A$ is a Cuntz-Krieger algebra.

**Corollary (A-Ruiz)**

Corners of Cuntz-Krieger algebras are Cuntz-Krieger algebras.
Extensions of purely infinite Cuntz-Krieger algebras

**Definition**

A $C^*$-algebra $A$ looks like a purely infinite Cuntz-Krieger algebra if

- $A$ is unital, purely infinite, nuclear, separable, and of real rank zero,
- $A$ has finitely many ideals,
- for all $I \subseteq J \subseteq A$, the group $K_*(J/I)$ is finitely generated, the group $K_1(J/I)$ is free, and $\text{rank } K_0(J/I) = \text{rank } K_1(J/I)$,
- the simple subquotients of $A$ are in the bootstrap class.

**Observation**

Consider a unital extension $I \hookrightarrow A \twoheadrightarrow B$. If $A$ is a purely infinite Cuntz-Krieger algebra, then

1. $B$ is a purely infinite Cuntz-Krieger algebra,
2. $I$ is stably isomorphic to a purely infinite Cuntz-Krieger algebra,
3. $K_0(B) \to K_1(I)$ vanishes.

If 1–3 holds, then $A$ looks like a purely infinite Cuntz-Krieger algebra.
Classification of purely infinite Cuntz-Krieger algebras

Theorem (Restorff)

Let $A$ and $B$ be purely infinite Cuntz-Krieger algebras with $\text{Prim}(A) \cong \text{Prim}(B)$. Then $\text{FK}_R(A) \cong \text{FK}_R(B)$ implies $A \otimes K \cong B \otimes K$.

Example (Reduced filtered $K$-theory $\text{FK}_R$)

For a $C^*$-algebra $A$ with ideal lattice

\[
\begin{array}{ccc}
A & \xrightarrow{K_0(I)} & K_0(J_n) \\
\downarrow & & \uparrow \\
J_1 & & K_0(J_n/I) \\
\downarrow & & \\
J_2 & & K_1(I), \\
\downarrow & & n \in \{1, 2\} \\
I & & \\
\downarrow & & \\
0 & & 
\end{array}
\]

its $\text{FK}_R(A)$ consists of $K_0(J_n/I)$.

Theorem (A-Bentmann-Katsura)

Let $A$ be a $C^*$-algebra that looks like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra $B$ with $\text{Prim}(A) \cong \text{Prim}(B)$ and $\text{FK}_R(A) \cong \text{FK}_R(B)$. 

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Applying classification to extensions of Cuntz-Krieger algebras

**Theorem (A-Bentmann-Katsura)**

Let $A$ be a $C^*$-algebra that look like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra $B$ with $\text{Prim}(A) \cong \text{Prim}(B)$ and $\text{FK}_R(A) \cong \text{FK}_R(B)$.

**Theorem (Kirchberg, Meyer-Nest, Bentmann-Köhler)**

Let $A$ and $B$ be Kirchberg $X$-algebras with $X$ an accordion space. Then $\text{FK}_K(A) \cong \text{FK}_K(B)$ implies $A \otimes K \cong B \otimes K$.

**Theorem (A-Bentmann-Katsura)**

Let $A$ and $B$ be $C^*$-algebras that looks like purely infinite Cuntz-Krieger algebras and assume that $\text{Prim}(A)$ and $\text{Prim}(B)$ are homeomorphic accordion spaces. Then $\text{FK}_R(A) \cong \text{FK}_R(B)$ implies $\text{FK}(A) \cong \text{FK}(B)$.

**Corollary**

Let $A$ be a $C^*$-algebra with $\text{Prim}(A)$ an accordion space. Then $A$ is a purely infinite Cuntz-Krieger algebra if and only if it looks like one.