Closure properties for the class of Cuntz-Krieger algebras

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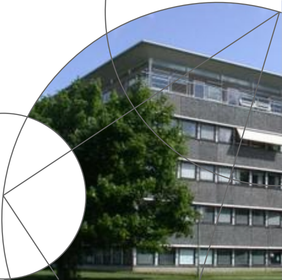

Corners of Cuntz-Krieger algebras

Theorem (A-Ruiz)

Let E be a countable directed graph. TFAE:

- $C^*(E)$ *is a Cuntz-Krieger algebra,*
- E *is finite with no sinks,*
- $C^*(E)$ *is unital and* $\operatorname{rank} K_0(C^*(E)) = \operatorname{rank} K_1(C^*(E))$.

Theorem (A-Ruiz)

Let A be a unital C^*-algebra and assume that A is stably isomorphic to a Cuntz-Krieger algebra. Then A is a Cuntz-Krieger algebra.

Corollary (A-Ruiz)

Corners of Cuntz-Krieger algebras are Cuntz-Krieger algebras.

Extensions of purely infinite Cuntz-Krieger algebras

Definition

A C^*-algebra A *looks like a purely infinite Cuntz-Krieger algebra* if

- A is unital, purely infinite, nuclear, separable, and of real rank zero,
- A has finitely many ideals,
- for all $I \trianglelefteq J \trianglelefteq A$, the group $K_*(J/I)$ is finitely generated, the group $K_1(J/I)$ is free, and $\operatorname{rank} K_0(J/I) = \operatorname{rank} K_1(J/I)$,
- the simple subquotients of A are in the bootstrap class.

Observation

Consider a unital extension $I \hookrightarrow A \twoheadrightarrow B$. If A is a purely infinite Cuntz-Krieger algebra, then

1. B is a purely infinite Cuntz-Krieger algebra,
2. I is stably isomorphic to a purely infinite Cuntz-Krieger algebra,
3. $K_0(B) \to K_1(I)$ vanishes.

If 1–3 holds, then A looks like a purely infinite Cuntz-Krieger algebra.

Classification of purely infinite Cuntz-Krieger algebras

Theorem (Restorff)

Let A and B be purely infinite Cuntz-Krieger algebras with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$. Then $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$ implies $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

Example (Reduced filtered K-theory $\mathrm{FK}_{\mathcal{R}}$)

For a C^*-algebra A with ideal lattice

$$\begin{array}{ccccc} & & A & & \\ & / & & \backslash & \\ J_1 & & & & J_2, \\ & \backslash & & / & \\ & & I & & \\ & & \vert & & \\ & & 0 & & \end{array}$$

its $\mathrm{FK}_{\mathcal{R}}(A)$ consists of

$$\begin{array}{ccc} K_0(I) & \longrightarrow & K_0(J_n) \\ \uparrow & & \\ K_0(J_n/I) & & \end{array} \qquad K_1(I),$$

$$n \in \{1, 2\}.$$

Theorem (A-Bentmann-Katsura)

Let A be a C^*-algebra that looks like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra B with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$ and $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$.

Applying classification to extensions of Cuntz-Krieger algebras

Theorem (A-Bentmann-Katsura)

Let A be a C^*-algebra that look like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra B with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$ *and* $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$.

Theorem (Kirchberg, Meyer-Nest, Bentmann-Köhler)

Let A and B be Kirchberg X-algebras with X an accordion space. Then $\mathrm{FK}(A) \cong \mathrm{FK}(B)$ *implies* $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

Theorem (A-Bentmann-Katsura)

Let A and B be C^*-algebras that looks like purely infinite Cuntz-Krieger algebras and assume that $\mathrm{Prim}(A)$ *and* $\mathrm{Prim}(B)$ *are homeomorphic accordion spaces. Then* $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$ *implies* $\mathrm{FK}(A) \cong \mathrm{FK}(B)$.

Corollary

Let A be a C^*-algebra with $\mathrm{Prim}(A)$ *an accordion space. Then A is a purely infinite Cuntz-Krieger algebra if and only if it looks like one.*