Bachelor projects for mathematics and mathematics-economics

Department of Mathematical Sciences
University of Copenhagen

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Contents

Introduction

This is a catalogue of projects suggested by the researchers at the Department for Mathematical Sciences for students in the B.S. programs in mathematics and mathematics-economics. It is important to note that such a catalogue will never exhaust all possibilities – indeed, if you are not finding what you are looking for you are strongly encouraged to ask the member of our staff you think is best qualified to help you on your way for suggestions of how to complement what this catalogue contains. Also, the mathematics-economics students are encouraged to study the searchable list of potenial advisors at the Economy Department on

`www.econ.ku.dk/polit/undervisning_og_opgaver/speciale/vejlederoversigt/`.

If you do not know what person to approach at the Department of Mathematical Sciences, you are welcome to try to ask

- the director of studies (Ernst Hansen, erhansen@math.ku.dk) or
- the assocate chair for education (Jesper Lützen, lutzen@math.ku.dk).

When you have found an advisor and agreed on a project, you must produce a contract (your advisor will know how this is done), which must then be approved by the director of studies at the latest during the first week of a block. The project must be handed in during the 7th week of the following block, and an oral defense will take place during the ninth week.

We wish you a succesful and engaging project period!

Best regards,

Jesper Lützen	Ernst Hansen
Associate chair	Director of studies

1 Finance

1.1 Rolf Poulsen

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Relevant interests:

Finance.

Suggested projects:

- Option pricing [Fin1]
  Pricing and hedging of exotic options (barrier, American, cliquet). A detailed investigation of convergence in of the binomial model. Multi-dimensional lattices. Model calibration as an inverse problem.

- Stochastic interest rates [Fin1]
  Yield curve estimation. Estimation of dynamic short rate models. Calibration and the forward algorithm. Derivative pricing with applications to embedded options in mortgage products, the leveling algorithm. Risk management for mortgagors and pension funds.

- Optimal portfolio choice [Fin1]
  Quadratic optimization with linear but non-trivial constraints. Multi-period optimal portfolio choice via stochastic programming. An experimental approach to equilibrium.

- Model risk [Fin1]
  `http://www.math.ku.dk/~rolf/Fin2_2010.doc`

Previous projects:

- Pricing of cliquet option [Fin1]
- The Black-Litterman Model [Fin1]
- The critical line algorithm and beyond [Fin1]
- Financial networks and systemic risk [Fin1]

1.2 Other projects

Other projects in this area can be found with

- Jens Hugger (4.3)

2 Operations research

2.1 Trine K. Boomsma and Salvador Pineda

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Relevant interests:

Mathematical programming, stochastic programming, dynamic programming, real options, energy applications.

Suggested projects:

- Multi-stage Stochastic Programming vs. Stochastic Dynamic Programming [OR1 + one of the following courses: Advanced mathematical programming, OK, GAMS, OR2, Stochastic Programming]
  Stochastic programming is a mathematical framework that allows to solve optimization problems when some of the parameters involved are not fully known. For example, stochastic programming can be used to address both everyday issues such as "*which road should I take to get to work without knowing the traffic level in each of them?*" and complicated problems such as *determining the optimal portfolio of assets in a market without knowing how prices will evolve.*

  However, the decisions made in the two previous examples are not final, but can be modified as uncertain information discloses. For example, if we decide to drive to work through road A and we realize after some kilometres that the traffic is very high, we may be able to switch to road B. Likewise, if we observe high prices of a given asset, we will try to increase the share of such asset to improve our final profit. Multi-stage stochastic programming succeeds to solve problems in which decisions may change as uncertain information becomes available. However, these type of problems present the curse of dimensionality, which basically means that the number of equations and variables drastically increase with the number of decision stages.

  Alternatively, these problems can be solved using dynamic programming, which is a methodology that requires that decisions at each stage are discretized. For example, in the case of the assets, we should impose that assets can be sold or bought in lots of 10. By doing so, we can obtain the solution to the initial problem by solving smaller subproblems. While dynamic programming reduces the computational burden of some models, it does not provide the exact optimal solution because of the discretization of the decision variables.

  Within this framework, this project will focus on comparing these two mathematical tools and determine in which cases multi-stage stochastic programming overcomes dynamic programming and vice versa. To evaluate each methodology in a real-life problem, we propose an energy planning model that aims at determining the electricity production of different types of generating units facing the uncertainty of electricity demand level and wind power production.

3 Algebra and number theory

3.1 Henrik G. Holm

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Relevant interests:

Rings, modules, homological algebra, category theory.
The prerequisites for the following projects are the courses [Alg 1] and [Alg 2]. Details, and possibly additional suggestions for projects, may be found at my homepage `http://www.math.ku.dk/~holm/`

Suggested projects:

- Completion of rings [Alg1, Alg2]
  Given an ideal I in a commutative ring R one can construct the I-adic completion $\widehat{R}_I$. For example, $\widehat{k[x]}_{(x)}$ is the formal power series ring and $\widehat{\mathbb{Z}}_{(p)}$ is the ring of p-adic integers. The aim of this project is to define adic completions and to investigate their basic properties.
  Literature: H. Matsumura, "Commutative ring theory".

- Group (co)homology [Alg1, Alg2]
  To a group G one can associate a sequence of (abelian) homology groups $H_n(G)$ and cohomology groups $H^n(G)$ that contain information about G. For example, $H_1(G) = G_{\mathrm{ab}}$ is the abelianization of G. The aim of this project is to define group (co)homology and to give group theoretical descriptions of the lower (co)homology groups.
  Literature: P. J. Hilton and U. Stammbach, "A course in homological algebra".

- Gröbner bases [Alg1, Alg2]
  Given an ideal I in the polynomial ring $k[x_1, \ldots, x_n]$ and a term ordering $\leqslant$ one can always find a so-called Gröbner basis $g_1, \ldots, g_m$ of I with respect to $\leqslant$. For example, a Gröbner basis for the ideal $I = (y^2 - x^3 + x, y^3 - x^2)$ with respect to the lexicographic ordering (where $x \geqslant y$) consists of $g_1 = y^9 - 2y^6 - y^4 + y^3$ and $g_2 = x - y^7 + y^4 + y^2$. Gröbner bases are powerful tools to solve e.g. polynomial equations and the ideal membership problem. The aim of this project is to define, and to show the existence of, Gröbner bases, and to demonstrate some applications.
  Literature: N. Lauritzen, "Concrete abstract algebra" and D. Cox, J. Little, and D. O'Shea, "Ideals, varieties, and algorithms".

- Injective modules [Alg1, Alg2]
  An object in a category is called injective if it has a certain lifting property. For example, the injective objects in the category of abelian groups are precisely the divisible abelian groups (such as the group of rational numbers $\mathbb{Q}$ and the Prüfer groups $\mathbb{Z}(p^\infty)$ where p is a prime). The aim of this project is to develop the theory of injective modules over an arbitrary ring.
  Literature: F. W. Anderson and K. R. Fuller, "Rings and categories of modules" and E. E. Enochs and O. M. G. Jenda, "Relative homological algebra".

- The lower K-groups of a ring [Alg1, Alg2]
  The algebraic K-theory of a ring R is a certain sequence $K_n(R)$ of abelian groups that contains information about R. For example, if R is a field, then $K_0(R) = \mathbb{Z}$ is the additive group of integers and $K_1(R) = R^\times$ is the multiplicative group of units in R. The aim of this project is to define and investigate the lower K-groups for certain classes of rings.
  Literature: J. Rosenberg, "Algebraic K-theory and its applications".

3.2 Christian U. Jensen

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Relevant interests:

Galois theory. Algebraic number theory.

Suggested projects:

- Introductory Galois theory [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics, but is also a very rich field that can be studied in its own right.

- Introduction to algebraic number theory [Alg2]
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

3.3 Ian Kiming

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Relevant interests:

Algebraic number theory and arithmetic geometry.

Suggested projects:

- Introduction to algebraic number theory [Alg2]
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and

the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

- First case of Fermat's last theorem for regular exponents [Alg2]
  The project studies the proof of Fermat's last theorem for 'regular' prime exponents p in the so-called first case: this is the statement that $x^p + y^p + z^p = 0$ does not have any solutions in integers x, y, z not divisible by p. The project involves studying some introductory algebraic number theory which will then also reveal the definition of 'regular primes'.

- p-adic numbers [Alg2]
  The real numbers arise from the rational numbers by a process called 'completion'. It turns out that the rational numbers (and more generally any algebraic number field) has infinitely many other 'completions', namely one associated to each prime number p. The fields that arise in this way are called the fields of p-adic numbers. They have a lot of applications in many branches of mathematics, not least in the theory of Diophantine equations, i.e., the question of solving in integers polynomial equations with integral coefficients.

- Hasse–Minkowski's theorem on rational quadratic forms [Alg2]
  A rational quadratic form is a homogeneous polynomial with rational coefficients. The Hasse–Minkowski theorem states that such a polynomial has a non-trivial rational zero if and only if it has a non-trivial zero in the real numbers and in all fields of p-adic numbers. The latter condition can be translated into a finite number of congruence conditions modulo certain prime powers and thus one obtains an effective criterion. The project involves an initial study of p-adic numbers.

- Continued fractions and Pell's equation [Alg2]
  The project studies the theory of continued fractions and how this can be applied to determining units in quadratic number rings. This has applications to the study of Pell (and 'non-Pell') equations, i.e., solving equations $x^2 - Dy^2 = \pm 1$ in integers for a given positive, squarefree integer D.

- Class groups of quadratic number fields and binary quadratic forms [Alg2]
  A quadratic number field is a field obtained from $\mathbb{Q}$ by adjoining a number of form $\sqrt{D}$ where D is an integer that is not a square (in $\mathbb{Z}$.) The class group attached to such a field measures how far its so-called ring of integers in from being a unique factorization domain. These class groups are necessary to study of one wants to understand integer solutions to equations of form $ax^2 + by^2 = c$ for given integers a, b, c.

- Modular forms on $\mathrm{SL}_2(\mathbb{Z})$ [Alg2, KomAn]
  This project studies modular forms on $\mathrm{SL}_2(\mathbb{Z})$. These are initially analytic objects and thus a certain, minimal background in complex analysis is required.

Modular forms turn out to have a lot of deep connections to arithmetic, and one can use this project as a platform for a later study of the more general modular forms on congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$. These are very important in modern number theory and are for instance central in Andrew Wiles' proof of Fermat's last theorem.

- Introductory Galois theory [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics.

- Group cohomology [Alg2]
  Group cohomology is a basic and enormously important mathematical theory with applications in algebra, topology, and number theory. The project will study the initial theory staring with cohomology of discrete groups and then perhaps move on to cohomology of profinite groups. This project can be used as a platform for continuing with study of Galois cohomology and Selmer groups.

- The theorem of Billing–Mahler [Alg2, EllKurv]
  A big theorem of Barry Mazur (1977) implies in particular that if n is the order of a rational point of finite order on an elliptic curve defined over $\mathbb{Q}$ then either $1 \leq n \leq 10$ or $n = 12$. Thus, in particular, $n = 11$ is impossible. This latter statement is the theorem of Billing and Mahler (1940). The project studies the proof of the theorem of Billing–Mahler which will involve a bit more theory of elliptic curves as well as an initial study of algebraic number theory. The impossibility of $n = 13$ can also be proved with these methods.

- Torsion points on elliptic curves [Alg2, EllKurv]
  The project continues the study of elliptic curves defined over $\mathbb{Q}$ in the direction of a deeper study of (rational) torsion points. There are several possibilities here, for instance, parametrizations of curves with a point of a given, low order, generalizations of the Nagell–Lutz theorem, the structure of the group of torsion points on elliptic curves defined over a p-adic field (Lutz' theorem).

- Primality testing [Alg2]
  How can one decide efficiently whether a large number is a prime number? The project will study one or more of the mathematically sophisticated methods of doing this: the Miller–Rabin probabilistic primality test and/or the more recent Agrawal-Kayak-Saxena deterministic primality test. The project will include an initial study of algorithmic complexity theory.

- Factorization algorithms [Alg2]
  How can one find the prime factorization of a large number? The project will study one or more of the mathematically sophisticated methods of doing this: the Dixon factorization method, factorization via continued fractions, the quadratic sieve. The project will include an initial study of algorithmic complexity theory.

- Open project [?]
  If you have some ideas on your own for a project within the general area of number theory, you can always come and discuss the possibilities with me.

Previous projects:

- The Agrawal-Kayak-Saxena primality test [Alg2]
- Selmer groups and Mordell's theorem [Alg3, EllKurv]
- Hasse–Minkowski's theorem on rational quadratic forms [Alg2]
- Torsion points on elliptic curves [Alg2, EllKurv]
- Factorization via continued fractions [Alg2, Krypto]
- The Pohlig-Hellman algorithm for computing discrete logarithms [Alg2]
- Schoof's algorithm [Alg3, EllKurv]

3.4 Jørn B. Olsson

`olsson@math.ku.dk`

Relevant interests:

Finite groups and their characters, finite symmetric groups and related topics from combinatorics and number theory

Suggested projects:

- Results on Permutation groups [Alg 2]
  Give a thorough description of selected abstract results on permutation groups, supplemented by concrete explicit examples.
  Literature: H. Kurzweil-B. Stellmacher, Theory of finite groups / D. Passman, Permutation groups
- Some properties of finite solvable groups [Alg 2]
  There is a number of interesting results on finite solvable groups, which helps you understand some of their characteristic properties, for instance a generalization of Sylow's theorem. The purpose of the project is to present some of these results.
  Literature: D.J.S. Robinson, A Course in the Theory of Groups / M. Hall, Theory of Groups
- Some finite p-groups [Alg 2]
  A p-group is a group of prime power order. Such groups have a rich structure and there are many of them. Present some basic results and a number of concrete examples.

- Equations in finite groups [Alg 2]
We consider equations on the form $x^n = c$, where c is an element of a finite group G. Give a proof of Frobenius' theorem on the number of solutions to such an equation and study more explicitly the case, where G is a symmetric group.

- Generators and relations in groups [Alg 2]
Give a description of a free group and explain how a group may be defined by generators and relations on the generators. This should be illustrated by concrete examples.
Literature: D.J.S. Robinson, A Course in the Theory of Groups / M. Hall, Theory of Groups

- Groups of small order [Alg 2, Alg 3]
Present some basic tools to study groups of a given finite order and apply them to "classify" groups of relatively small order.

- Integer partitions []
There is a very extensive literature on integer partitions. They play a role in representation theory, in combinatorics and in number theory. Present some examples of simple basic results on partitions, based primarily on the book by Andrews and Eriksson and illustrate the results by examples.
Literature: G.E Andrews - K. Eriksson, Integer Partitions

- The Robinson-Schensted correspondence and its properties [Alg 2]
The Robinson-Schensted correspondence is an interesting natural bijection between the set of permutations and the set of pairs of so-called standard tableaux of the same shape, which is fairly easy to describe. Present the the definition of a standard tableau and of the Robinson-Schensted correspondence and illustrate some of its basic properties.
Literature: B. Sagan, The Symmetric Group

- Standard tableaux and the hook formula [Alg 2]
This project is of a combinatorial nature and of relevance for the representation theory of symmetric groups. The surprisingly nice hook formula tells you what the number of standard tableux of given shape is. Present the definitions of partitions, of hooks in partitions, of standard tableaux of a given (partition) shape and prove the brancing rule for standard tableaux and then the hook formula, using an inductive argument. Illustrate with explicit examples.

- Specht modules for symmetric groups [Alg 2]
This is a basic construction in representation theory of symmetric groups. Give a brief introduction to the group algebra and its modules and describe the irreducible modules in the case of the symmetric groups.
Literature: B. Sagan, The Symmetric Group

3.5 Ellen Henke

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Relevant interests:

Group theory and saturated fusion systems.

Suggested projects:

- AN EXPLANATION BEFOREHAND []
  The first five project are about the classification of finite simple groups (short: CFSG) and about saturated fusion systems. Many other projects about these topics are possible! The CFSG is one of the most celebrated theorems of 20th century mathematics. The proof is very long, probably too long for any mathematician to understand completely. However, there are some principal ideas in the proof one can try to understand. Principally, one first classifies the structure of the p-local subgroups, i.e. the normalizers of the non-trivial p-subgroups. One distinguishes two cases (mainly for the prime $p = 2$): The first case is that the finite group G is of characteristic p-type, which means that for any p-local subgroup M of G, there is a normal p-subgroup P of M with $C_M(P) \leq P$ (where $C_M(P) := \{g \in M : gx = xg$ for all $x \in P\})$. The second case is that G is of so-called "component type" meaning basically that it is not of characterstic p-type. Saturated fusion systems model the p-local structure of finite groups, so many ideas from the CFSG work also in this generalized setting.

- THE GOLDSCHMIDT AMALGAM METHOD []
  This is one of the main tools for an elegant treatment of groups of characteristic p-type. The idea is to use a graph Γ, constructed from two subgroups M_1, M_2 of G with certain properties. The vertices of Γ are the right cosets of M_1 and M_2 in G, and two vertices are joint by an edge if they intersect non-trivially. See Kurzweil, Stellmacher, The Theory of Finite Groups, Chapter 10.3.

- SIGNALIZER FUNCTORS: []
  Signalizer functors are the main tool to treat groups of component type. Understand the definition and some important results like for example the Completeness Theorem of Glauberman. See Kurzweil, Stellmacher, Chapter 11.

- N-GROUPS: []
  Understand how Signalizer functor methods and the amalgam method work together by looking at an example as given in Kurzweil, Stellmacher, Chapter 12.

- FUSION SYSTEMS ON SMALL p-GROUPS: []
  As saturated fusion systems generalize the p-local structure of a finite group, every finite group leads to a saturated fusion system. However, there are saturated fusion systems which do not come from finite groups and these are called exotic. It is not very well understood at the moment when such exotic examples occur, but to find examples at least in small cases, it is often useful to classify fusion systems attached to a concretely chosen finite p-group of relatively small order. As a project, get familiar with the definition of saturated fusion systems, look

at an example of how other people classified fusion systems on a small p-group, pick your favorite p-group and try to do the same yourself!

- Transporter systems and localities []
  To saturated fusion systems one can add an additional structure, namely a transporter system (as defined by Oliver-Ventura) or equivalently, a locality (as defined by Andrew Chermak). A special case are centric linking systems, which are particularly important to study the homotopy theory of fusion systems. As a project, understand the definitions of saturated fusion systems, transporter systems/localities and centric linking systems, play around with the definitions and try to construct some interesting new examples.

- Applications of character theory to finite groups []
  The proofs of some theorems about finite groups use character theory. A character of a finite group G is obtained as follows: Fix a field k and a group homomorphism ϕ from G to $GL_n(k)$. Then $\chi : G \to k$ defined by $\chi(g) := trace(\phi(g))$ is a character. Most of the proofs of group theoretical results which use character theory work with the field $k = \mathbb{C}$. Get familiar with the basics of the theory for $k = \mathbb{C}$, and look at some applications like for example at the proof of Burnsides $p^a q^b$-Theorem, or at the Brauer–Suzuki Theorem in the case that a Sylow 2-subgroup has order 8.

- The automorphism tower problem []
  This is a project of a somewhat different flavor, since it is at the intersection of group theory and set theory. Let G be a group with $Z(G) = 1$. Define a chain of groups by setting $G_0 := G$ and $G_{i+1} := Aut(G_i)$ for $i \geq 0$. Does this chain ever become stationary? If $|G|$ is finite, the chain becomes stationary after finitely many steps by a classical result of Wieland. If $|G|$ is infinite there is a result of Thomas saying that the chain becomes stationary as well, but not necessarily after finitely many steps! Get familiar with enough group theory and set theory to understand Wieland's and Thomas' proofs of these facts, write an exposition about that.

3.6 Søren Jøndrup

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Relevant interests:

Ikke kommutativ ringteori, ikke kommutativ affin algebraisk geometri, automorfigrupper for ringe.

Suggested projects:

- Identiteter for matricer [Alg2]
  Det er let at se, at for 3 2×2 matricer A, B og C med koefficienter i et legeme gælder:

$$(AB - BA)^2 C - C(AB - BA)^2 = 0$$

Vi diskuterer sådanne "tilsvarende" resultater for $n \times n$-matricer og giver et bevis ved hjælp af grafteori for den såkaldte Amitsur-Levitzki's sætning. Denne sætning siger, at der findes en identitet af grad $2n$ for $n \times n$-matricer. Endvidere indses, at $n \times n$-matricer ikke opfylder nogen identitet af lavere grad. Anvendelser på ringteori diskuteres.

- Den fri gruppe og den fri algebra med anvendelser [Alg2]
Det er kendt, at Diedergruppen D_n er frembragt af 2 elementer r og s, som opfylder:

$$r^n = s^2 = 1 \text{ og } rs = sr^{-1}$$

Vi diskuterer, hvad det vil sige, at det er alle relationer i D_n. Endvidere sætter vi disse resultater ind i en mere generel ramme og Todd-Coxeter algoritmen behandles. Tilsvarende resultater for ringe undersøges.

3.7 Other projects

Other projects in this area can be found with

- Christian Berg (4.1)
- Jesper Grodal (7.1)
- Morten S. Risager (4.5)

4 Analysis

4.1 Christian Berg

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Relevant interests:

Orthogonal polynomials and moment problems. Complex analysis. Commutative harmonic analysis.

Suggested projects:

- The Gamma function [An1,KomAn]
  Euler's Gamma function is the most import of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.
- Entire functions [An1, Koman]
  Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.
- Fibonacci numbers [An1]
  The Fibonacci numbers 0,1,1,2,3,5,... are determined by taking the sum of the previous two numbers to get the next. They occur in many different areas of mathematics and have interesting number theoretical properties. Furthermore they have connections to the theory of orthogonal polynomials, cf. `www.math.ku.dk/~berg/manus/normathilbert.pdf`.

Previous projects:

- Spherical functions [An1]
- Conformal mapping [An1, KomAn]
- Topological groups, Haar measure [An1,MI]

4.2 Bergfinnur Durhuus

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Relevant interests:

Analysis: Operator theory, differential equations. Mathematical physics: Quantum mechanics, statistical mechanics. Discrete mathematics: Graph theory, analytic combinatorics, complexity theory,

Suggested projects:

- Graph colouring problems [Dis1, An1]
  Problems originating from various areas of mathematics can frequently be formulated as colouring problems for certain types of graphs. The four-colour problem is probably the best known of coulouring problems but there is a variety of other interesting colouring problems to attack

- Combinatorics of graphs [Dis1, An1, ComAn]
  Counting of graphs specified by certain properties (e.g. trees) is one of the classical combinatorial problems in graph theory having applications in e.g. complexity theory. The method of generating functions is a particularly effective method for a large class of such problems making use of basic results from complex analysis

- Unbounded opreators and self-adjointness [An2]
  Many of the interesting operators playing a role in mathematical physics, in particular differential operators of use in classical and quantum mechanics, are unbounded. The extension of fundamental results valid for bounded operators on a Hilbert space, such as the notion of adjoint operator and diagonalisation properties, is therefore of importance and turns out to be non-trivial

Previous projects:

- Clifford algebras, Spin groups and Dirac operators [Alg1,An2]
- Ramsey theory [Dis1,An1]
- Causal Structures [An1,Geom2]
- The Tutte polynomial [Dis1,An1]
- Knot theory and statistical mechanics [Dis1,AN1]
- Graph 3-colourings [Dis1,An1]
- Minimal surfaces [Geom1,An1]
- Planar graphs [Dis1,AN1]

4.3 Jens Hugger

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Relevant interests:

Numerical analysis – eScience

Suggested projects:

- NUMERICAL METHODS FOR PRICING OPTIONS [NumIntro, preferably also NumDiff]
  Pick one or more options and "solve" it with one or more numerical methods. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the option, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- NUMERICAL METHODS FOR OPTIMIZATION [NumIntro, preferably also NumDiff]
  Pick an optimization problem and "solve" it with one or more numerical methods. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the optimization problem, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS [NumIntro, NumDiff]
  Pick a differential equation and solve it with a numerical method. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the problem, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- CONVERGENCE OF NUMERICAL METHODS FOR PDE'S [An2]
  Learn the theory of convergence analysis for numerical methods for PDE's. Apply the theory to a real life problem (of your choice or provided by me like for example the Asian option from finance theory.

- NUMERICAL METHODS FOR INTERPOLATION OR INTEGRATION IN SEVERAL DIMENSIONS OR ITERATIVE SOLUTION OF LARGE EQUATION SYSTEMS [NumIntro]
  Pick a problem and solve it with a numerical method. Either bring your own problem or get one from the advisor.

- PORTING PART OF A MAPLE PROGRAM INTO A FAST PROGRAMMING LANGUAGE [NumIntro, Programming experience with Maple and a fast programming language like Fortran, C etc]
  Replace the slow part of a Maple code for solving an Asian option with code written in a faster language. Document your code.

Previous projects:

- CONVECTION-DIFFUSION IN ONE VARIABLE [NumDiff]
- ASIAN OPTIONS [NumDiff]

4.4 Henrik L. Petersen

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Relevant interests:

Suggested projects:

- Picard's theorems [KomAn, some measure theory]
  If you are presented with an entire function f and you have two different complex numbers not in the image set $f(\mathbb{C})$ then f is constant. This result is known as Picard's little theorem.

- Riemann's mapping theorem [KomAn]
  Any simply connected region $\mathcal{D}$ in the complex plane except the plane itself is conformally equivalent to the open unit disk Δ, meaning that there exists a holomorphic and bijective mapping $\varphi : \Delta \to \mathcal{D}$.

- Müntz-Szasz' theorem [KomAn, Functional Analysis]
  Let $\{\lambda_k\}$ be an increasing sequence of positive numbers. When is the span of the power-functions $\{1, x^{\lambda_1}, x^{\lambda_2}, \cdots\}$ dense in the space of continuous functions on $[0, 1]$? Answer: exactly when $\sum_{k=1}^{\infty} 1/\lambda_k = \infty$!

- Paley–Wiener's theorem [KomAn, some measure theory]
  The Fourier transform $\widehat{\phi}$ of a function ϕ from the Hilbert space $L_2(-a, a)$ can be extended to an entire function of exponential type (meaning that its growth is dominated by $e^{K|z|}$ for all large $|z|$. Conversely, any entire function of enxponential type is in fact the Fourier transform of an L_2-function of a finite interval.

4.5 Morten S. Risager

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Relevant interests:

Number theory, automorphic forms, complex analysis, Riemann surfaces.

Suggested projects:

- The prime number theorem [KomAn, An2]
  The prime number theorem gives a quantitative version of Euclid theorem about the infinitude of primes: it describes how the primes are distributed among the integers. It was conjectured 100 years before the first proof.

- Twin primes and sieve theorems [KomAn, An2]
  Very little is known about the number of twin primes. Using sieve methods one can show that the sum of reciprocicals of twin primes is convergent. Still it is not known if there are only finitely many or not.

- THE FUNCTIONAL EQUATION FOR RIEMANN'S ZETA FUNCTION [KomAn, An2]
  Using methods from Fourier analysis - in particular Poisson summation - one investigates the properties of Riemann's famous zeta function.

- COUNTING ELEMENTS IN FREE GROUPS [KomAn, An2]
  How does one count in a resonable way the number of elements in the free group on n generators? Using methods from linear algebra one can give good asymptotic and statistical results. Numerical investigations is also a possibility.

Previous projects:

- ELEMENTARY METHODS IN NUMBER THEORY, AND A THEOREM OF TERRENCE TAO. [An2, ElmTal]

- PRIMES IN ARITHMETIC PROGRESSIONS [KomAn, An2]

- SMALL EIGENVALUES OF THE AUTOMORPHIC LAPLACIAN AND RADEMACHERS CONJECTURE FOR CONGRUENCE GROUPS [KomAn, An3]

4.6 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- THE HEISENBERG GROUP [An1,An2]
  The Heisenberg group is important, for example because it is generated by the position and momentum operators in quantum mechanics. The purpose of this project is to study its representation theory. A famous theorem of Stone and von Neumann relates all irreducible representations to the Schrödinger representation acting on $L^2(\mathbf{R}^n)$.

- UNCERTAINTY PRINCIPLES [An1,Sand1,KomAn]
  Various mathematical formulations of the Heisenberg uncertainty principle are studied. Expressed mathematically, the principle asserts that a non-zero function f on $\mathbf{R}$ and its Fourier transform $\hat{f}$ cannot be simultaneously concentrated. A precise version, called the Heisenberg inequality, expresses this in terms of standard deviations. A variant of the theorem, due to Hardy, states that f and $\hat{f}$ cannot both decay more rapidly than a Gaussian function.

- THE PETER-WEYL THEOREM [An1,An2,Sand1]
  The purpose of this project is to study $L^2(G)$ for a compact group G, equipped with Haar measure. The theorem of Peter and Weyl describes how this space

can be orthogonally decomposed into finite dimensional subspaces, which are invariant under left and right displacements by G. Existence of Haar measure can be proved or assumed.

5 Geometry

5.1 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- GLOBAL PROPERTIES OF CURVES (AND/OR SURFACES) [Geom1,An1]
  The differential geometry studied in Geometry 1 is of a local nature. The curvature of a curve in a point, for example, describes a property of the curve just in the vicinity of that point. In this project the focus is on *global* aspects of closed curves, as for example expressed in Fenchel's theorem, which gives a lower bound for the total integral of the curvature, in terms of the perimeter.

- GEODESIC DISTANCE [Geom1,An1]
  The geodesic distance between two points on a surface is the shortest length of a geodesic joining them. It turns the surface into a metric space. The project consists of describing some propreties of the metric. For example Bonnet's theorem: *If the Gaussian curvature is everywhere* ≥ 1, *then all distances are* $\leq \pi$.

5.2 Hans Plesner Jakobsen

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Relevant interests:

Unitaritet, Liegrupper, Liealgebraer, kvantiserede matrixalgebraer, kovariante differentialoperatorer i matematisk fysik, kvantiserede indhyldningsalgebraer

Suggested projects:

- SYMMETRIER [ca. 1 øars matematik]
  - Diskrete symmetrier: Tapetgrupper, Krystallografiske grupper.
  - Kontinuerte symmetrier: Rotationsgruppen, Lorentzgruppen, PoincarÈgruppen, den konforme gruppe,...

- GAUSS-BONNET [Forudsætter ca. 1 års matematik]
  Den måske mest fundamentale sætning i Euklidisk geometri er Thales' sætning, der siger, at summen af vinklerne i en trekant er $180°$. Denne sætning kan generaliseres til trekanter pâglatte flader i rummet. Herved fremkommer Gauss-Bonnet's sætning i lokal udgave. Den globale udgave af sætningen leder til en invariant for kompakte flader: Eulerkarakteristikken.

- De kanoniske kommutatorrelationer [ca. 1 års matematik + Hilbertrum]
  Operatorerne Q og P givet ved $(QF)(x) = xF(x)$ of $(PF)(x) = -i(\frac{dF}{dx})(x)$ er forbundet via Fouriertransformationen, men kan Fouriertransformationen 'konstrueres' ud fra disse? Kan man bygge en bølgeoperator eller en Diracoperator ud af den harmoniske oscillator?

- Liealgebraer [ca. 1 års matematik + (kan aftales)]
  (F.eks.) Klassifikation. Dynkin diagrammer, Kac-Moody algebraer, super Liealgebraer. Hvad fik Borcherds (bl.a.) Fieldsmedaljen for?

- Matrix Liegrupper [ca. 1 års matematik]
  Bl.a. eksponentialfunktionen for matricer, tensorprodukter, duale vektorrum. Er der en forbindelse mellem Peter Weyl Sætningen og Stone Weirstrass Sætningen?

5.3 Other projects

Other projects in this area can be found with

- Ib Madsen (7.2)
- Nathalie Wahl (7.4)

6 Noncommutativity

6.1 Erik Christensen

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Relevant interests:

Group Algebras, Non Commutative Geometry, Fractal Sets, Convexity, Operator Algebras.

Suggested projects:

- Discrete groups and their operator algebras [Analysis 3]
  Many aspects of discrete groups are reflected in the operator algebras generated by their unitary representations.

- Elementary aspects of Non Commutative Geometry applied to Fractal Sets [Analysis 3]
  Even though fractal sets are quite far from being smooth, it is possible to describe parts of the geometry of a Cantor set or the Sierpinski Gasket using tools from non commutative geometry

- Convexity and Discrete Geometry [Analysis 1]
  Convex sets have many nice properties and the methods used fit quite naturally with familiar arguments from the plane or the 3-dimensional space. There is a lot of difficult problems which may be reached even for a bachelor student.

- Exercises on Operator Algebra [Analysis 3]
  Based on the course Analysis 3 you may want to learn more on certain aspects of operator algebras. This project consists in reading a text and demonstrating your understanding of the items read by solving several exercises.

6.2 Søren Eilers

eilers@math.ku.dk

Relevant interests:

Advanced linear algebra related to operator algebras. Dynamical systems. Mathematics in computer science; computer science in mathematics.

Suggested projects:

- Perron-Frobenius theory with applications [LinAlg, An1]
  Methods involving matrix algebra lead to applications such as Google's PageRank and to the ranking of American football teams.

- Data storage with symbolic dynamics [An1, Dis1]
  Engineering constraints neccessitate a recoding of arbitary binary sequences into sequences meeting certain constraints such as "between two consecutive ones are at least 1, and at most 3, zeroes". Understanding how this is done requires a combination of analysis and discrete mathematics involving notions such as entropy and encoder graphs.

- Experimental mathematics [LinAlg]
  Design an experiment in Maple to investigate a mathematical problem, cf. `www.math.ku.dk/~eilers/xm`.

Previous projects:

- An experimental approach to flow equivalence [An1]
- Visualization of non-euclidean geometry [MatM, Geom1]
- Planar geometry in high school mathematics [MatM]
- Liapounov's theorem [MI]

6.3 Niels Grønbæk

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Relevant interests:

Banachrum, banachalgebra, kohomologi, matematikkens didaktik

Suggested projects:

- Et undervisningsforløb på gymnasialt niveau [LinAlg, An1, Alg1, Geo1]
  Projektet går ud på at tilrettelægge, udføre og evaluere et undervisningsforløb af ca. 2 ugers varighed i en gymnasieklasse.

Suggested projects:

- Amenable Banach Algebras [An3]
  Amenability of Banach algebras is an important concept which originates in harmonic analysis of locally compact groups. In the project you will establish this connection and apply it to specific Banach algebras such as the Banach algebra of compact operators on a Hilbert space.

6.4 Magdalena Musat

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Relevant interests:

Banach Spaces, Functional Analysis, Operator Algebras, Probability Theory

Suggested projects:

- Geometry of Banach spaces [Analysis 3]
  A number of very interesting problems concerning the geometry of Banach spaces can be addressed in a bachelor project. For example, does every infinite dimensional Banach space contain an infinite dimensional reflexive subspace or an isomorphic copy of l_1 or c_0? Or, does there exist a reflexive Banach space in which neither an l_p-space, nor a c_0-space can embed? Another project could explore the theory of type and cotype, which provides a scale for measuring how close a given Banach space is to being a Hilbert space.

- Convexity in Banach spaces [Analysis 3]
  The question of differentiability of the norm of a given Banach space is closely related to certain convexity properties of it, such as uniform convexity, smoothness and uniform smoothness. This project will explore these connections, and study further properties of uniformly convex (respectively, uniformly smooth) spaces. The Lebesgue spaces L_p $(1 < p < \infty)$ are both uniformly convex and uniformly smooth.

- Haar measure [MI]
  This project is devoted to the proof of existence and uniqueness of left (respectively, right) Haar measure on a locally compact topological group G. For example, Lebesgue measure is a (left and right) Haar measure on $\mathbb{R}$, and counting measure is a (left and right) Haar measure on the integers (or any group with the discrete topology).

- Fernique's theorem [SAND 1, Analysis 3]
  This project deals with probability theory concepts in the setting of Banach spaces, that is, random variables taking values in a (possibly infinite dimensional) Banach space. Fernique's theorem generalizes the result that gaussian distributions on $\mathbb{R}$ have exponential tails to the (infinite dimensional) setting of gaussian measures on arbitrary Banach spaces.

6.5 Søren Knudby

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Relevant interests:

Topological groups, measure theory, functional analysis, differential algebra

Suggested projects:

- Differential algebra [Alg2, KomAn]
  The purpose of this project is to develop the theory of differential algebra and to prove Liouville's Theorem concerning the existence of elementary antiderivatives of elementary functions. As an application it will be possible to prove (and give a precise meaning to the statement) that e^{-x^2} has no antiderivative expressible using only elementary functions.

- Topological groups and Haar measure [A2, MI, Top]
  The project should contain an introduction to the theory of topological groups. The main goal is hereafter to prove the existence and uniqueness of the Haar measure on a locally compact group. Examples of locally compact groups are the Euclidean spaces or the circle with Lebesgue measure. The project should include a proof of the Riesz representation theorem which gives a correspondence between measure on the one hand and linear functionals on the other hand.

- Locally compact abelian groups [A2, MI, Top]
  To any locally compact abelian group G is associated a dual group $\hat{G}$ which is also locally compact and abelian. For instance, the dual group of the integers is the circle group, and the dual group of the circle group is the group of integers. There is an intimate connection between G and its dual group $\hat{G}$. This is displayed in Bochner's Theorem which relates (positive definite) functions on G to measures on $\hat{G}$. The ultimate result in this area is Pontryagin's duality theorem which identifies the dual group of $\hat{G}$ with the original group G itself.

- Compact groups [A2, MI, Top]
  The project concerns basic representation theory of compact groups. The purpose is to study the Hilbert space $L^2(G)$ of a compact group G, equipped with Haar measure. The main goal is the Peter-Weyl Theorem which describes how the space $L^2(G)$ decomposes into orthogonal finite dimensional subspaces, which are invariant under left and right translations by elements of G.

6.6 Ryszard Nest

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Relevant interests:

Non-Commutative Geometry, Deformation Theory, Poisson Geometry

Suggested projects:

- Clifford Algebras [LinAlg, Geom 1]
  Clifford algebra is a family $\mathcal{C}^{p,q}$ of finite dimensional algebras associated to non-degenerate bilinear forms which play very important role in both topology and geometry. The simplest examples are $\mathbb{R}$, $\mathbb{C}$ and the quaternion algebra $\mathbb{H}$. The main result is the periodicity modulo eight of $\mathcal{C}^{p,q}$, which has far reaching

consequences (e.g., Bott periodicity, construction of Dirac operators) in various areas of mathematics.

- Axiom of choice and the Banach-Tarski paradox [LinAlg, Analysis 1]
  The axiom of choice, stating that for every set of mutually disjoint nonempty sets there exists a set that has exactly one member common with each of these sets, is one of the more "obvious" assumptions of set theory, but has far reaching consequences. Most of modern mathematics is based on its more or less tacit assumption. The goal of this project is to study equivalent formulations of the axiom of choice and some of its more exotic consequences, like the *Banach-Tarski paradox*, which says that one can decompose a solid ball of radius one into five pieces, and then rearrange those into two solid balls, both with radius one.

- Formal deformations of $\mathbb{R}^{2n}$ [LinAlg, Geom 1]
  The uncertainty principle in quantum mechanics says that the coordinate and momentum variables satisfy the relation $[p, x] = \hbar$, where $\hbar$ is the Planck constant. This particular project is about constructing associative products in $C^\infty(\mathbb{R}^{2n})[[\hbar]]$ satisfying this relation and studying their properties.

6.7 Mikael Rørdam

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Relevant interests:

Operator Algebras, Topics in Measure Theory, Discrete Mathematics

Suggested projects:

- Topics in C^*-algebras [Analysis 3]
  C^*-algebras can be defined either abstractly, as a Banach algebra with an involution, or concretely, as subalgebras of the algebra of bounded operators on a Hilbert space. They can be viewed as non-commutative analogues of spaces, since every commutative C^*-algebra is equal to the set of continuous functions on a locally compact Hausdorff space. Several topics concerning C^*-algebras and concerning the study of specific examples of C^*-algebras, can serve as interesting topics for a bachelor project.

- Topics in measure theory [MI]
  We can here look at more advanced topics from measure theory, that are not covered in MI, such as existence (and uniqueness) of Lebesgue measure, or more generally of Haar measure on locally compact groups. Results on non-measurability are intriguing, perhaps most spectacularly seen in the Banach-Tarski paradox that gives a recipe for making two solid balls of radius one out of a single solid ball of radius one!

- Topics in discrete mathematics [Dis2 & Graf]
  One can for example study theorems about coloring of graphs. One can even combine graph theory and functional analysis and study C^*-algebras arising from graphs and the interplay between the two (in which case more prerequisites are needed).

Previous projects:

- Irrational and rational rotation C^*-algebras [Analyse 3]
- Convexity in functional analysis [Analyse 3]
- The Banach-Tarski Paradox [MI recommended]

6.8 Thomas Vils Petersen

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Relevant interests:

Functional analysis, analysis, Banach algebras of functions.

Suggested projects:

- Convolution algebras [An2, MI]
  These are Banach algebras of functions, and the product is the convolution product. Some possible topics:
  - Derivations on $L^1[0, 1]$.
  - Homomorphisms between weighted convolution algebras $L^1(\omega)$ on the half-line $\mathbb{R}^+$.

7 Topology

7.1 Jesper Grodal

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Relevant interests:

Topology, Algebra, Geometry.

Suggested projects:

- Group cohomology [Alg2]
  To a group G we can associate a collection of abelian groups $H^n(G)$, $n \in \mathbf{N}$, containing structural information about the group we started with. The aim of the project would be to define these groups, examine some of their properties, and/or examine applications to algebra, topology, or number theory. See e.g.: K.S. Brown: Cohomology of groups

- Group actions [Top, Alg2]
  How can groups act on different combinatorial or geometric objects? Eg. which groups can act freely on a tree? See e.g.: J.-P. Serre: Trees.

- The Burnside ring [Alg2]
  Given a group G we can consider the set of isomorphism classes of finite G-sets. These can be "added" and "multiplied" via disjoint union and cartesian projects. By formally introducing additive inverses we get a ring called the Burnside ring. What's the structure of this ring and what does it have to do with the group we started with? See:
  http://en.wikipedia.org/wiki/Burnside_ring

- The classification of finite simple groups [Alg2]
  One of the most celebrated theorems in 20th century mathematics gives a complete catalogue of finite simple groups. They either belong to one of three infinite families (cyclic, alternating, or classical) or are one of 26 sporadic cases. The aim of the project is to explore this theorem and perhaps one or more of the sporadic simple groups. See:
  http://en.wikipedia.org/wiki/Classification_of_finite_simple_groups

- The Platonic solids and their symmetries [Top, Alg2]
  A Platonic solid is a convex polyhedron whose faces are congruent regular polygons, with the same number of faces meeting each vertex. The ancient greeks already knew that there were only 5 platonic solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The aim of the project is to understand the mathematics behind this. See: http://en.wikipedia.org/wiki/Platonic_solid

- TOPOLOGICAL SPACES FROM CATEGORIES [Top, Alg2]
  Various algebraic or combinatorial structures can be encoded via geometric objects. These "classifying spaces" can then be studied via geometric methods. The goal of the project would be to study one of the many instances of these this, and the project can be tilted in either topological, categorical, or combinatorial directions. See e.g.: A. Bj?rner, Topological methods. Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.

- SIMPLICIAL COMPLEXES IN ALGEBRA AND TOPOLOGY [Alg1, Top]
  The goal of this project is to understand how simplicial complexes can be used to set up a mirror between notions in topology and algebra. For instance, the algebraic mirror image of a topological sphere is a Gorenstein ring.

Previous projects:

- STEENROD OPERATIONS—CONSTRUCTION AND APPLICATIONS [AlgTopII]
- HOMOTOPY THEORY OF TOPOLOGICAL SPACES AND SIMPLICIAL SETS [AlgTopII]
- AUTOMORPHISMS OF G - WITH APPLICATIONS TO GROUP EXTENSIONS [AlgTopII, CatTop]

7.2 Ib Madsen

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Relevant interests:

Homotopy theory, topology of manifolds.

Suggested projects:

- DE RHAM COHOMOLOGY []
- POINCARÉ DUALITY []
- COVERING SPACES AND GALOIS THEORY []
- THE HOPF INVARIANT []

7.3 Jesper Michael Møller

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Relevant interests:

All kinds of mathematics.

Suggested projects:

- POINCARÉ SPHERE [Topology, group theory]
  What are the properties of the Poincaré sphere?
- TOPOLOGICAL COMBINATORICS [Dis1, Top]
  Combinatorial problems, such as determining chromatic numbers of graphs, can be solved using topological methods.
- PARTIALLY ORDERED SETS [Dis1]
  Partially ordered sets are fundamental mathematical structures that lie behind phenomena such as the Principle of Inclusion-Exclusion and the Möbius inversion formula.
- CHAOS [General topology]
  What is chaos and where does it occur?
- PROJECT OF THE DAY [Mathematics]
  `http://www.math.ku.dk/~moller/undervisning/fagprojekter.html`

7.4 Nathalie Wahl

`wahl@math.ku.dk`

Relevant interests:

Graphs, surfaces, 3-dimensional manifolds, knots, algebraic structures.

Suggested projects:

- KNOTS [Alg1,Top]
  Mathematically, knots are embeddings of circles in 3-dimensional space. They are rather complicated objects that can be studied combinatorially or via 3-manifolds. The project consists of learning some basics in knot theory. See for example `http://www.earlham.edu/~peters/knotlink.htm`.
- BRAID GROUPS, CONFIGURATION SPACES AND LINKS [Alg1,Top]
  The braid group on n strands can be defined in terms of braids (or strings), or as the fundamental group of the space of configurations of n points in the plane. It is related to knots and links, and also to surfaces. The project consists of exploring braid groups or related groups like mapping class groups. See for example J. Birman, Braids, links, and mapping class groups.

- Classification of surfaces [Top,Geom1]
  Closed 2-dimensional surfaces can be completely classified by their genus (number of holes). There are several ways of proving this fact and the project is to study one of the proofs. See for example W. Massey, A Basic Course in Algebraic Topology, or A. Gramain, Topology of Surfaces.

- 3-manifolds [Top,Geom1]
  3-dimensional manifolds are a lot harder to study than 2-dimensional ones. The geometrization conjecture (probably proved recently by Perelman) gives a description of the basic building blocks of 3-manifolds. Other approaches to 3-manifolds include knots, or "heegaard splittings", named after the Danish mathematician Poul Heegaard. The project consists of exploring the world of 3-manifolds. See for example `http://en.wikipedia.org/wiki/3-manifolds`.

- Non-Euclidean geometries [Geom1]
  Euclidean geometry is the geometry we are used to, where parallel lines exist and never meet, where the sum of the angles in a triangle is always 180°. But there are geometries where these facts are no longer true. Important examples are the hyperbolic and the spherical geometries. The project consists of exploring non-euclidian geometries. See for example
  `http://en.wikipedia.org/wiki/Non-euclidean_geometries`

- Frobenius algebras, Hopf algebras [LinAlg,Alg1]
  A Frobenius algebra is an algebra with extra structure that can be described algebraically or using surfaces. A Hopf algebra is a similar structure. Both types of algebraic structures occur many places in mathematics. The project consists of looking at examples and properties of these algebraic structures. See for example J. Kock, Frobenius algebras and 2D topological quantum field theories.

- Khovanov Homology [AlgTop – or familiarity with category theory]
  The complexity of knots is immense. Explore `http://katlas.org/`. Over the last 100 years various tools have been developed to distinguish and classify knots. A lot of work is still needed to have a good understanding of the world of knots.

  This project would aim at understanding one of the stronger tools available to this date; Khovanov Homology.

- Operads and Algebras [Alg2]
  Operads is an effective tool to cope with exotic algebraic structures. How do you for instance work with algebraic structures that are not (strictly) associative? A framework for given a broader perspective on various types of algebras would be developped. Depending on interest, pointers towards geometric and topological algebraic aspects is also a possibility. Nathalie Wahl is also a potential supervisor on this project.

- Morse Theory [Geom2 – for instance simultaneously]
  The 2. derivative test, known from MatIntro, tells you about local characteristica of a 2-variable function. Expanding this test to manifolds in general yields Morse Theory, which plays a key role in modern geometry.

This project would start out by introducing Morse Theory. Various structure and classification results about manifolds could be shown as applications of the theory.

8 History and philosophy of mathematics

8.1 Jesper Lützen

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Relevant interests:

History of Mathematics

Suggested projects:

- The history of non-Euclidean geometry [Hist1, preferrably VtMat]
  How did non-Euclidean geometry arise and how was its consistency "proved". How did the new geometry affect the epistemology of mathematics?
- The development of the function concept [Hist1]
  How did the concept of function become the central one in mathematical analysis and how did the meaning of the term change over time.
- Archimedes and his mathematics [Hist1]
  Give a critical account of the exciting life of this first rate mathematician and analyze his "indivisible" method and his use of the exhaustion method.
- What is a mathematical proof, and what is its purpose [Hist1, VtMat]
  Give philosophical and historical accounts of the role(s) played by proofs in the development of mathematics

Previous projects:

- A brief history of complex numbers [Hist1, preferrably KomAn]
- Mathematical induction. A history [Hist1]
- Aspects of Euler's number theory [Hist1, ElmTal]
- Mathematics in Plato's dialogues [Hist1, VtMat]
- Axiomatization of geometry from Euclid to Hilbert [Hist1, preferrably VtMat]
- Lakatos' philosophy applied to the four color theorem [Dis, Hist1]
- History of mathematics in mathematics teaching: How and why [Hist1, DidG preferrably DidMat]

9 Set Theory

9.1 Asger Törnquist

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Relevant interests:

Mathematical logic, descriptive set theory, ergodic theory, operator algebras.

Suggested projects:

- Classical and effective descriptive set theory []
  Classical descriptive set theory is concerned with the basic properties of sets and functions that arise in analysis, e.g. Borel sets. Effective descriptive set theory is a modern refinement of this theory which combines ideas from computability theory with the classical ideas. It is a powerful tool which has been used to prove surprising structure theorems about the structure of Borel equivalence relations and Borel graphs. A possible goal for a project could be to prove Silver's dichotomy theorem for Borel equivalence relations, which says that any Borel equivalence on $\mathbb{R}$ either must have countably many classes, or there must be a homeomorphic copy of the Cantor set which meets every class in *at most* one point.

  [The project requires some basic knowledge of metric spaces and general topology. Some knowledge of measure theory is desirable.]

- Axiomatic set theory and Gödel's constructible universe []
  This project is about the Zermelo-Fraenkel system of axioms for set theory (ZFC), which many today accept as the basic framework for all of mathematics. A goal of the project could be to introduce Gödel's constructible universe L, and use this to prove the following: If ZFC is consistent (that is, one cannot deduce a contradiction from it), then so is ZFC in conjunction with the Continuum Hypothesis ($2^{\aleph_0} = \aleph_1$.) This means that the continuum hypothesis cannot be disproved from ZFC (if ZFC is consistent), and cannot introduce a contradiction, if there isn't one there already.

- Ergodic theory and orbit equivalence []
  Ergodic theory is concerned with the study of measure preserving actions of groups on measure spaces. Each such action induces an orbit equivalence relation (Danish: Baneækvivalensrelation). It is of great interest to study how much the orbit equivalence relation in itself "remembers" about the group and the action. A goal of the project could be to prove Dye's theorem: All "ergodic" (that is, irreducible, in some sense) actions of $\mathbb{Z}$ on the unit interval $[0, 1]$ which preserve Lebesgue measure produce essentially the same equivalence relation, up to a natural notion of isomorphism known as *orbit equivalence*. A more ambitious goal would be to also prove Hjorth's theorem, which stands in sharp contrast to Dye's: Every countably infinite discrete group with *property (T)* (e.g. $\mathrm{SL}_3(\mathbb{Z})$) has uncountably many orbit *in*equivalent measure preserving ergodic actions on $[0, 1]$.

[The project requires some knowledge of measure theory, of metric spaces, and, for Hjorth's theorem, some elementary knowledge of operators on Hilbert spaces.]

10 Mathematical Models in Systems Biology

10.1 Elisenda Feliu

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Relevant interests:

Applied algebraic methods, Mathematics applied to biochemical processes, Computer implementation, ODEs

Suggested projects:

- MATHEMATICAL ANALYSIS OF BIOCHEMICAL SYSTEMS AT STEADY STATE (EQUILIBRIUM). [Alg2]
  The project consists of a theoretical part and an applied part. The theoretical part consists in learning how to describe a biochemical systems with ODEs and learning algebraic methods to understand biochemical reaction network models at steady state. The applied part of the project uses the techniques to a specific biochemical system (that will be chosen from the literature together with the student), and subsequent interpretation of the results. Application of the techniques involves the use of available software platforms and possibly implementation of small scripts in Maple.

10.2 Carsten Wiuf

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Relevant interests:

Markov chains applied to biochemical processes, Simulation, ODEs.

Suggested projects:

- MATHEMATICAL ANALYSIS OF BIOCHEMICAL SYSTEMS AT AND NEAR EQUILIBRIUM. [Diff, Stok]
  The project consists of a theoretical part and an applied part. The theoretical part consists in learning how to describe a system as a Markov chain (stochastic) and a system of ODEs (deterministic). The applied part of the project analyzes a biochemical system as a Markov chain and an ODE system and contrasts the results with each other. The biochemical system will be chosen from the literature together with the student. The applied part involves implementation of small scripts in R or Maple.

11 Other areas

11.1 Discrete mathematics

Projects in this area can be found with

- Bergfinnur Durhuus (4.2)
- Søren Eilers (6.2)
- Jørn B. Olsson (3.4)
- Mikael Rørdam (6.7)

11.2 Teaching and didactics in mathematics

Projects in this area can be found with

- Niels Grønbæk (6.3)
- Jesper Lützen (8.1)

11.3 Aspects of computer science

Projects in this area can be found with

- Søren Eilers (6.2)
- Jens Hugger (4.3)