Københavns Universitet
Eksamen ved Det naturvidenskabelige Fakultet sommer 1999

Matemtaik 3 GE

Written exam, 4 hours. All course material is allowed during the exam (alle sædvanlige hjælpemidler er tilladt).
There are 5 problems divided into 12 questions. All questions are given approximately the same weight.

Problem 1

T is a geodesic triangle on a sphere of radius 2 with the sum of internal angles equal to 2π. Find the area of T.

Problem 2

Let γ denote the circle on the unit sphere

$$S^2 = \{(x,y,z)|x^2+y^2+z^2=1\}$$

given by the intersection of S^2 with the plane $\{(x,y,z)|x=a\}$.

1° Find the geodesic curvature of γ.

2° Find the area of the subset of S^2 given by

$$\{(x,y,z)\in S^2|x\geq a\}.$$

Problem 3

Let

$$x:\{(u,v)|1<v<u<2\}\to\mathbb{R}^3$$

be a global parametrisation of a regular surface S such that the first fundamental form is given by

$$E=G=u,\ F=v.$$

Find the area of S.

/LK Opgavesættet fortsættes på side 2

Problem 4

Let S be a surface in $\mathbb{R}^3$ given by the image of the parametrisation x:

$$\{(u,v)|u>0, v>0\} \ni (u,v) \to x(u,v) = (u^2, uv, v^2) \in \mathbb{R}^3.$$

1° Show that S is a regular surface.
2° Show that, for each point $p \in S$, there exists a straight line $l \subset \mathbb{R}^3$ such that

$$l \cap S \text{ containes an interval with p in its interior.}$$

3° Find the Gauss curvature of S.
4° Find the first fundamental form of S.
5° Find the unit normal vector field N such that x the parametrisation is orientation preserving.
6° Find the second fundamental form of S at the point $x(1,1)$.
7° Find the principal curvatures of S at the point $x(1,1)$.

Problem 5

Let $\gamma :]a,b[\ni t \to \gamma(t) \in S$ be a geodesic on a regular surface S. Show that there are constants c_1, c_2 such that

$$t = c_1 s + c_2,$$

where s is an arclength parameter of γ.