Københavns Universitet
Eksamen ved Det naturvidenskabelige Fakultet vinter 2000-2001

Matematik 3GE

Written exam, 4 hours. All course material is allowed during the exam (alle sædvanlige hjælpemidler er tilladt).
There are 3 problems divided into 11 questions. All questions are given approximately the same weight.

Problem 1

(a) **Find** the regular points of the curve

$$\gamma(t) = (\frac{1}{2}t^2, t^3, \frac{1}{2}t^4)$$

(b) **Find** the curvature and torsion of γ in its regular points.

Problem 2

Let S denote the surface of revolution given by rotating the parabola $y = x^2 - 4$ around the X-axis.

(a) **Find** the part S_{reg} of S which consists of regular points.
(b) **Find** the principal curvatures and the principal directions at the point $(0, 4, 0)$.
(c) **Give** a set of local parametrisations of S_{reg} which cover the whole surface.
(d) **Find the first and the second fundamental form** of S_{reg} in terms of the chosen parametrisation.
(e) **Prove** that S_{reg} is diffeomorphic but not isometric to an open subset of the plane $\{(x, y, 0) \mid x, y \in \mathbb{R}\}$.

Problem 3

let $\mathcal{O}$ be an oriented simple region on a sphere of radius 1. We denote the boundary of $\mathcal{O}$ by $\mathcal{G}$ and give it the induced orientation. We assume that $\mathcal{G}$ is a simple (no selfintersections) piecewise smooth closed curve formed by joining n distinct points by geodesic arcs.

(a) **Find** the supremum of possible sums of the internal angles of $\mathcal{G}$
(b) Suppose that $\mathcal{G}'$ is another simple piecewise smooth closed curve obtained by joining by geodesic intervals n distinct points of the sphere. We assume that $\mathcal{G}'$ is entirely contained in $\mathcal{O}$.
Prove that the sum of internal angles of $\mathcal{G}'$ is smaller than the one for $\mathcal{G}$.
(c) **Give** a best lower bound on the sum of internal angles of $\mathcal{G}$.
(d) **Prove** that the lowest bound is never attained.