Copenhagen University, Institute of mathematics.
Naturvidenskabelig Embedseksamen, Matematik 3NA, Numerisk Analyse.
September 1993. Open book exam. All 10 questions are weighted evenly (10%). A grade is given only after all programming assignments have been turned in and accepted.

Problem 1: Systems of nonlinear equations and matrixnorms.

a) Starting from $x^{(1)}$, perform a Newton iteration on the system of equations $F(x) = 0$ (that is find $x^{(2)}$) where

$$x,\ x^{(1)},\ x^{(2)} \in \mathcal{R}^2, \quad F(x) = \begin{pmatrix} x_1^3 - x_2 \\ x_2^2 \end{pmatrix}, \quad x^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

b) Determine the condition number in the norm $\|\cdot\|_\infty$ for the Jacobi-matrix for the vector function F in the point $x^{(1)}$ with the notation given in (a).

c) The following three errors are given

$$e_1 = \|x^{(1)} - x^*\|_2 = 1.41, \quad e_2 = \|x^{(2)} - x^*\|_2 = 0.55, \quad e_3 = \|x^{(3)} - x^*\|_2 = 0.10$$

They represent the standard Euclidian norm of the error in three steps in a Newton iteration, x^* being the exact solution of $F(x) = 0$. Determine the order of convergence of the iteration based on these numbers.

Problem 2: Systems of linear equations.

a) Determine a Doolittle LU-decomposition for the matrix

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 6 & 7 \\ 0 & 12 & 18 \end{pmatrix}.$$

b) Solve using LU-decomposition and a forward and a backward substitution the equation

$$Ax = \begin{pmatrix} 0 \\ 5 \\ 18 \end{pmatrix},$$

for $x \in \mathcal{R}^3$, and with A given in (a).

Problem 3: Interpolation.

A third degree polynomium $y = f(x)$ takes the following values: $f(0) = 15$, $f(3) = f(5) = 0$, $f(4) = -13$.

a) Determine by polynomial interpolation the function value in 1, f(1).

b) Determine the function expression in the subinterval $[0, 3]$ for the natural cubic spline interpolation S of the given data: $S(0) = 15$, $S(3) = S(5) = 0$, $S(4) = -13$.

c) Determine by interpolation with the natural cubic spline an approximation of the function value in 1, f(1), and explain why this number is different from the number from (a).

Problem 4: Numerical integration

a) Compute with the composite trapezoidal rule with two equally large subintervals (3 nodalpoints) an approximation to

$$\int_0^1 x^4 dx, \text{ hvor } x \in \mathcal{R}.$$

b) How many equally large subintervals were necessary in (a) for the absolute value of the error to be guaranteed less than 0.01?

(Exam finished).