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DATALOGI A(M)

Opgavesættet består af 6 opgaver og omfatter 3 sider + 3 sider dokumentation af NAG-rutinen F04ATF. Ved bedømmelsen vægtes besvarelsen af de enkelte opgaver som anført for hver enkelt opgave. Alle sædvanlige hjælpemidler,dvs. bøger, notater og lommeregnere kan medbringes.

Opgave 1. Vægt 15%

Bestem uddata fra kørsel af nedenstående PASCAL-Program (resultatet skal begrundes).

```
program opg1;
var x,y:integer;

procedure p(x:integer;var y:integer);
begin
   x:=x+2;
   y:=y+3;
   writeln(x:4,y:4);
end;

procedure q;
begin
   if y<6
      then begin
              p(x,y);
              q;
           end
      else p(y,x);
end;

begin
   x:=2;
   y:=2;
   p(x,y);
   writeln(x:4,y:4);
   q;
   writeln(x:4,y:4);
end.
```

Opgave 2. Vægt 30%

Reelle $m \times m$ matricer kan i PASCAL for $m = 1, 2, \ldots, 10$, repræsenteres ved data af type

```
MATRIX=array[1..10,1..10] of real;
```

Skriv et PASCAL-program, der indlæser et heltal m, kontrollerer at $1 \leq m \leq 10$, og stopper, hvis dette ikke er tilfældet. Ellers fortsættes med indlæsningen af to reelle $m \times m$-matricer A og B. Programmet skal dernæst beregne og udskrive produktmatricen $A'B$, hvor A' betegner den transponerede matrix til A.

Gør rede for, hvordan opgaven kan løses ved hjælp af MATLAB.

Giv en vurdering af PASCAL-programmets køretid udtrykt ved m.

Opgave 3. Vægt 15%

Et PASCAL-program skal indlæse en linie indeholdende et udtryk af formen

$$u_1 \, o_1 \, u_2 \, o_2 \cdots o_{n-1} \, u_n$$

hvor $u_1, u_2, \ldots, u_n$ $(n = 1, 2, \ldots)$ hver er et af tegnene 'a', 'b' eller 'c', og $o_1, o_2, \ldots, o_{n-1}$ hver er et af tegnene '+' eller '−'.

a) Opstil et syntaksdiagram til styring af denne indlæsning.

b) Skriv en procedure, der indlæser en linie og kontrollerer, at den er i overensstemmelse med syntaksdiagrammet. Hvis linien indeholder fejl i forhold til syntaksdiagrammet, skal der stoppes med en fejlmeddelelse. Ellers skal værdien af udtrykket beregnes for $a = 1, b = 2, c = 3$ og udskrives.

Blanktegn er tilladt og skal ignoreres.

Opgave 4. Vægt 10%

Gør rede for hvorledes man ved hjælp af REDUCE kan finde den funktion, der er bestemt ved

$$f''(x) = x^2 sin(2x) - cos(3x) + 1$$
$$f'(0) = 5 \; og \; f(0) = 7$$

Opgave 5. Vægt 15%

Et PASCAL-program indeholder erklæringerne

```
type Elementtype = record
                       Navn : array[1..25] of char;
                       Kontingent : real;
                       Betalt : Boolean;
                   end;
var Medlemsfil,Betalte,Restanter : file of Elementtype;
```

Skriv en procedure, der læser `Medlemsfil` og skriver to nye filer `Betalte` og `Restanter`. `Betalte` skal indeholde de uændrede poster for de medlemmer, der har betalt (`Betalt = true`), medens `Restanter` skal indeholde de øvrige, og her skal kontingentet forhøjes med 10%.

Opgave 6. Vægt 15%

Skriv et FORTRAN-program, der

1. Indlæser et heltal N.

2. Indlæser en $N \times N$-matrix A.

3. Indlæser en $N \times 1$-matrix B.

4. Løser ligningssystmet $AX = B$ ved hjælp af NAG-biblioteksrutinen F04ATF (dokumentation er vedlagt).

5. Udskriver løsningen X.

Programmet skrives til en enkelt nøjagtigheds implementering af F04ATF.

F04ATF – NAG FORTRAN Library Routine Document

NOTE: before using this routine, please read the appropriate implementation document to check the interpretation of ***bold italicised*** terms and other implementation–dependent details. The routine name may be precision–dependent.

1. Purpose

F04ATF calculates the accurate solution of a set of real linear equations with a single right hand side, Ax=b, by Crout's factorisation method.

2. Specification

```
      SUBROUTINE F04ATF (A, IA, B, N, C, AA, IAA, WKS1, WKS2,
     1   IFAIL)
C     INTEGER IA, N, IAA, IFAIL
C     real A(IA,N), B(N), C(N), AA(IAA,N), WKS1(N), WKS2(N)
```

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3. Description

Given a set of linear equations, Ax = b, the routine first decomposes A using Crout's factorisation with partial pivoting PA = LU, where P is a permutation matrix, L is lower triangular and U is unit upper triangular. An approximation to x is found by forward and backward substitution in Ly = Pb and Ux = y. The residual vector r = b – Ax is then calculated and a correction, d, to x is found by the solution of LUd = r. x is replaced by (x+d) and the process repeated until full machine accuracy is obtained. ***Additional precision*** accumulation of innerproducts is used throughout the calculation.

4. References

[1] WILKINSON, J.H. and REINSCH, C. Handbook for Automatic Computation. Volume II, Linear Algebra. Springer–Verlag, 1971, pp. 93–110.

5. Parameters

A – ***real*** array of DIMENSION (IA,p) where p ≥ N.

Before entry, A must contain the elements of the real matrix.

Unchanged on exit.

IA – INTEGER.

On entry, IA must specify the first dimension of array A as declared in the calling (sub)program.
IA ≥ N.

Unchanged on exit.

B – ***real*** array of DIMENSION at least (N).

Before entry, B must contain the elements of the right hand side. (See Section 11).

Unchanged on exit.

N – INTEGER.

On entry, N must specify the order of matrix A.

Unchanged on exit.

C – ***real*** array of DIMENSION at least (N).

On successful exit, C will contain the solution vector.

AA – ***real*** array of DIMENSION (IAA,q) where q ≥ N.

Used as working space.

On successful exit, AA will contain the LU decomposition.

IAA – INTEGER.

On entry, IAA must specify the first dimension of array AA as declared in the calling (sub)program.
IAA ≥ N.

Unchanged on exit.

WKS1 – ***real*** array of DIMENSION at least (N).

WKS2 – ***real*** array of DIMENSION at least (N).

Used as working space.

IFAIL – INTEGER.

On entry, IFAIL must be set to 0 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

Unless the routine detects an error (see next section), IFAIL contains 0 on exit.

6. Error Indicators and Warnings

Errors detected by the routine:–

IFAIL = 1

The matrix A is singular, possibly due to rounding errors.

IFAIL = 2

The matrix A is too ill-conditioned to produce a correctly rounded solution.

7. Auxiliary Routines

Details are distributed to sites in machine-readable form.

8. Timing

The time taken is approximately proportional to N^3.

9. Storage

There are no internally declared arrays.

10. Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see [1], page 107.

11. Further Comments

The routine **must not** be called with the same name for parameters B and C.

12. Keywords

Accurate Solution of Linear Equations,
Crout Factorisation,
Real Matrix,
Single Right Hand Side.

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13. Example

To solve the set of linear equations Ax = b where

$$A = \begin{pmatrix} .33 & 16 & 72 \\ -24 & -10 & -57 \\ -8 & -4 & -17 \end{pmatrix} \text{ and } b = \begin{pmatrix} -359 \\ 281 \\ 85 \end{pmatrix}$$

13.1. Program Text

WARNING: This **single precision** example program may require amendment for certain implementations. The results produced may not be the same. If in doubt, please seek further advice (see **Essential Introduction** to the Library Manual).

```
C     F04ATF EXAMPLE PROGRAM TEXT
C     NAG COPYRIGHT 1975
C     MARK 4.5 REVISED
C
      REAL A(5,5), B(5), C(5), AA(5,5), WKS1(18), WKS2(3)
      INTEGER NIN, NOUT, I, N, J, IA, IAA, IFAIL
      DATA NIN /5/, NOUT /6/
      READ (NIN,99999) (WKS1(I),I=1,7)
      WRITE (NOUT,99997) (WKS1(I),I=1,6)
      N = 3
      READ (NIN,99998) ((A(I,J),J=1,N),I=1,N), (B(I),I=1,N)
      IA = 5
      IAA = 5
      IFAIL = 1
      CALL F04ATF(A, IA, B, N, C, AA, IAA, WKS1, WKS2, IFAIL)
      IF (IFAIL.EQ.0) GO TO 20
      WRITE (NOUT,99996) IFAIL
      STOP
   20 WRITE (NOUT,99995) (C(I),I=1,N)
      STOP
99999 FORMAT (6A4, 1A3)
```

```
99998 FORMAT (3F5.0)
99997 FORMAT (4(1X/), 1H , 5A4, 1A3, 7HRESULTS/1X)
99996 FORMAT (25H0ERROR IN F04ATF IFAIL = , I2)
99995 FORMAT (10H0SOLUTIONS/(1H , F4.1))
      END
```

13.2. Program Data

```
F04ATF EXAMPLE PROGRAM DATA
   33   16   72
  -24  -10  -57
   -8   -4  -17
 -359  281   85
```

13.3. Program Results

```
 F04ATF EXAMPLE PROGRAM RESULTS

 SOLUTIONS
  1.0
 -2.0
 -5.0
```