

# Symbolic dynamics and homeomorphisms of the Cantor set

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## 1 Program

### 1.1 Open problems in symbolic dynamics

**Mike Boyle, University of Maryland**

This will be a series of eight lectures around some open problems in symbolic dynamics. I've chosen problems where there is something interesting and useful to learn about as context, and where it is practical to get some understanding of the situation in the time allotted. Most of the time will be spent on that. The lectures will cover most or all of the following, with statements of associated problems:

- Classification theory and problems for  $\mathbb{Z}$  shifts of finite type (shift equivalence, strong shift equivalence, gyration function, dimension representation, Wagoner's strong shift equivalence CW complex, counterexample to Williams' Shift Equivalence Conjecture, the positive rational shift equivalence conjecture and the Kim-Roush method of invariant tetrahedra)
- The Generalized Spectral Conjecture (a conjectured characterization of the stable algebraic structure of primitive matrices over a unital subring of the reals)
- One sided shifts: classifications and embeddings (an easy classification; a hard classification; an open problem; the embedding contrast for one and two sided SFTs)

- Expansive subdynamics of  $\mathbb{Z}^d$  actions
- Commuting SFTs and textile systems
- $\mathbb{Z}^d$  SFTs: the uncovered recursive landscape, following especially Hochman and Meyerovitch

## 1.2 Cantor minimal systems and their orbit structure

**Christian Skau, Norwegian University of Science and Technology**

A Cantor minimal system is a topological dynamical system  $(X, T)$ , where  $X$  is the Cantor set and  $T$  is a homeomorphism on  $X$  such that all the orbits are dense in  $X$ . Such systems have a universal property: Every minimal system  $(Y, S)$ , where  $Y$  is a compact metric space is a factor of a Cantor minimal system.

We will present complete invariants for the orbit structure of Cantor minimal systems. These invariants are of (ordered)  $K$ -theoretic nature and can be defined in purely dynamical terms, even though they are intimately related to the  $K_0$  group of the associated  $C^*$ -crossed product, and are so-called (simple) dimension groups. Bratteli diagrams, dynamically interpreted, will be the crucial tool for getting a grasp on the orbit structure. Important examples of Cantor minimal systems will be given, like odometers, substitution systems and Toeplitz systems.

We will then study the orbit structure of Cantor minimal  $\mathbb{Z}^d$  systems, i.e. a (minimal) action of the  $d$ -dimensional ( $d > 1$ ) free abelian group on  $d$  generators. In other words, we have  $d$  commuting homeomorphisms of the Cantor set such that all the orbits are dense. We will show that such a system is orbit equivalent to a Cantor minimal system, i.e. to a minimal  $\mathbb{Z}$ -action. A complete orbit invariant for such systems is again a simple dimension group.

Finally, we will list some challenging open problems in this area.

## 1.3 $C^*$ -algebras associated to symbolic dynamics

**Toke Meier Carlsen, University of Southern Denmark**

I plan to go through the definition of  $C^*$ -algebras associated to shift spaces as universal  $C^*$ -algebras and show that the  $C^*$ -algebra of a shift space is invariant up to one-sided conjugacy, and that the stabilized  $C^*$ -algebra of a shift space is invariant up to two-sided conjugacy and flow equivalence. I

will also look at the  $K$ -theory of the  $C^*$ -algebra of a shift space and show how this give rise to new (and some old) invariants for shift spaces, and I will look at some examples, such as shifts of finite type, sofic shifts, shift spaces of substitutions (there will be some connection with Cantor minimal systems here) and beta-shifts and discuss what we can learn about these shift spaces and their classification by studying the corresponding  $C^*$ -algebras.

I plan to organize my lectures so the can be followed without any previous knowledge about  $C^*$ -algebras.