The classification of the finite simple groups begins with the Gorenstein-Walter Dichotomy Theorem (cf. [ALSS]) which says that each finite group $G$ of 2-rank at least 3 is either of component type or of characteristic 2-type. In the case of a saturated fusion system $\mathcal{F}$ the situation is nicer and the corresponding result is much easier to prove: $\mathcal{F}$ is either of characteristic $p$-type or component type. There is a program in progress that attempts, first, to translate some of the mathematics used to classify the finite simple groups of component type into the category of saturated 2-fusion systems, and then, second, to classify the saturated simple 2-fusion systems of component type. Finally, third, one could attempt to use the classification of the fusion systems of component type to obtain a classification of the finite simple groups of component type. Hopefully this would provide a simplification of the proof of the theorem classifying the finite simple groups. In particular such an approach would avoid the necessity of classifying the unbalanced groups and providing a proof of the B-Conjecture.

I will begin my series of lectures with a discussion of groups of component type, including an outline of the classification of such groups. Then I will provide some background on fusion systems and the translation of local finite group theory into a local theory of fusion systems. Finally I’ll outline a program for classifying simple saturated 2-fusion systems of component type, indicating on the one hand which parts of the program are complete or nearly complete, and on the other, where important notions from local group theory have not as yet been translated into analogous notions for fusion systems, or the necessary theorems on fusion systems have not as yet been proved.

REFERENCES

My standard reference for finite group theory, and particularly for local group theory is:


A good reference for facts about the simple groups is:


For an exposition outlining the original proof of the classification of the finite simple groups, see:


Basic references on fusion systems include:

More technical papers on fusion systems include: