Group Actions - Problems (2013)

Review of 2011 problems

1. rand 2 finite gps act freely on $S^n \times S^m$?
   - $\mathbb{Qd}(p)$ not solved?
   - no equivariant proj. to $S^m$ for $\mathbb{Qd}(p)$
   - does $\mathbb{Qd}(p)$ act on a p-local $CN \simeq S^n \times S^m$?
   - $p = 3$ special? $\exists$ compact group with
     - subgroup $\mathbb{Qd}(3)$ (Adem-Singer)
     - $H^*(\mathbb{Qd}(p); \mathbb{Z})$

2. $X = S^n$, $G = \Sigma_p$ existence of smooth action?
   \[
   \{ \text{G-actn with effective Euler char} \} \rightarrow \exists \text{ finite } G \text{ act } X \simeq S^n
   \]

   Milnor condition? Necessary and Sufficient
   
   Smooth actn: $H \in \text{Top}(X)$ work
   
   => $H \not\in H/\{H\}$ no finite deduced subgroups.

3. $X = S^n$, $G = \Sigma_p$ existence of smooth action?

4. $X = S^n$, $G = \Sigma_p$ existence of smooth action?

5. $X = S^n$, $G = \Sigma_p$ existence of smooth action?

   J. Davis (smoothly)

(Adem) Does every compact Lie gp $G$ act
freely on some product of spheres, $E_g \in U(n)$
(7) What 2-group \( E \) of finite rank and degree on some \( X \cong S^n \times S^m \times \mathbb{R}^2 \)?
(what about co-compactly)?

(8) \( \text{rk}(G) = \text{rk}(\text{ker}(G)) \) ?

For \( G \) a p-group, rank 3, \( \text{rank} > 3 \) ?
smooth actions: 
- Yekin Yalin

rank 2 case, still open?

For \( G \) finite p-group, \( \exists \) finite \( G \)-CW \( X \)

a product of spheres, but not smooth actions?

- no known obstructions

(9) Carlson question: \( \sum_{i=0}^{2n} \text{dim} H^i(X, \mathbb{F}_p) > 2^n \)

\( G = \langle 2/p \rangle \times \text{finite, free } G \)-CW \( X \).

new cases of Conner conjecture

- O.B. Kister + Yekin: "high dim" cases

(10) \( H^0(\text{BAut}(X), \mathbb{Q}) \) of finite transcendence degree?
\( X \) finite 1-cmn. (of Berglund-Madsen)

(11) \( G \) finite action self on \( 2\)-cx \( X \), \( X \not\approx \emptyset \)

Does it have a fixed pt? (O.6. - Segeev)
2 Group Cohomology

14. Work by Benson - Carlson?
15. Benson - Vredal - Henke
   p odd \( G_1 \rightarrow G_2 \) same p-Sylow subgroup
   then iso on p-Sylow => use on mod p cohom variables

21. ask A. Putnam

23. J. Møller student \( G = S_n \) or \( A_n \)?