Open Problem Session

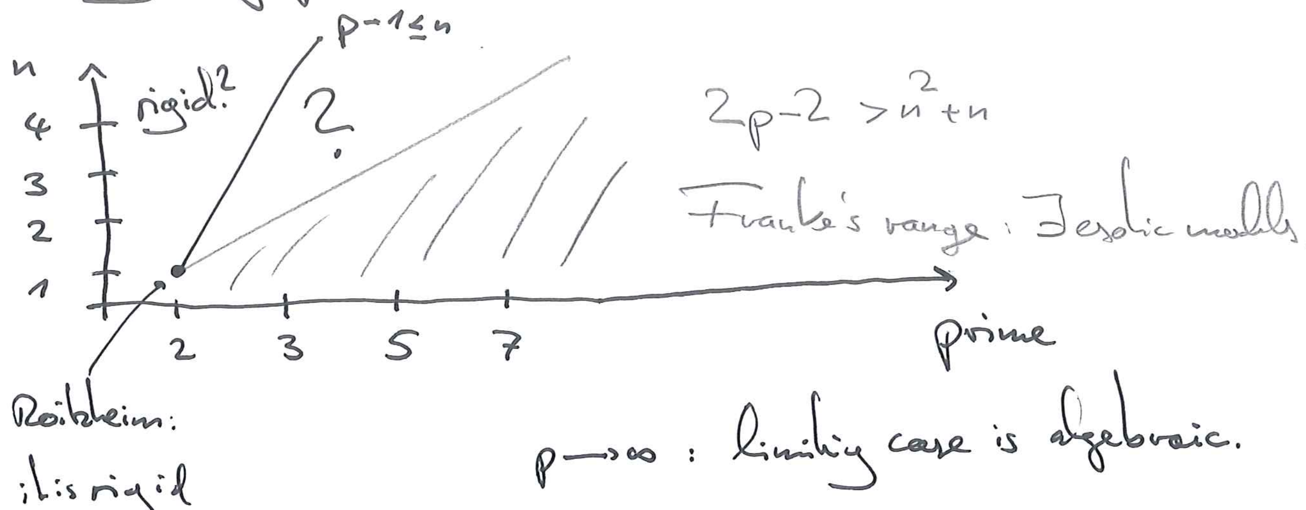
Schwede

Q. Is $Ho(MU\text{-mod})$ algebraic?

Q. Is there an exotic self-equivalence of Spectra?

Note: $Pic(Sp) \cong \mathbb{Z}$ gen by Σ.

Q. Rigidity for L_n-local / L_n^f-local.



$p \to \infty$: limiting case is algebraic.

$(\alpha_1 \beta_1^{p-1} \in \langle \underbrace{p, \beta_1, \dots, p, \beta_1, p}_{2p-1} \rangle$

Strickland

Q. Combine Tambara theory with alg models.

Finite G, finite G-set X

$\mathcal{C}(X) =$ X parametrized G-spectra

$f: X \to Y$ gives $f^*: \mathcal{C}(Y) \to \mathcal{C}(X)$

$N_f, T_f: \mathcal{C}(X) \to \mathcal{C}(Y)$

"category-valued Tambara functor".

$\mathcal{V}(X) =$ eq bundles of graded $\mathbb{Q}$-space over X.

probably: cartesian fibrations over ∞-category of bispans.

<u>Guess:</u> $\mathcal{C} \simeq \mathcal{V}$.

<u>Pakhkovia</u>

p odd ($p \geq 5$ for triangulation)

<u>Franke:</u> $L_1 Sp_{(p)}$ $\quad X \xrightarrow{\sim} Y \iff K_*(X) \otimes \mathbb{Z}_{(p)} \xrightarrow{\simeq} K_*(Y) \otimes \mathbb{Z}_{(p)}$.

Bousfield $\quad E(1)_*E(1)$ comodules $\quad M \longrightarrow M \otimes_{E(1)_*} E(1)_*E(1)$, $K_{(p)} \simeq \bigvee_{i=0}^{p-2} \Sigma^{2i} E(1)$.

$$D^{(1,[1])}(E(1)_*E(1)-\text{comods}) \underset{\Delta}{\simeq} Ho(L_1 Sp_{(p)}).$$

(gl dim $= 2$)

$M \xrightarrow{d} M[1]$

$d^2 = 0$

(for $p=3$, only an equivalence.)

$\hookrightarrow$ also: not monoidal.

G finite $\quad p \nmid |G|$

$$L_1 Sp_{G,(p)} \not\simeq \prod_{(H) \leq G} L_1 Sp_{(p)}[W_G(H)].$$

Greenlees-May, Kedziorek, Barnes

$$Ho(L_1 Sp_{(p)}[W_G(H)]) \underset{\Delta}{\simeq} Ho(E(1)_*E(1)-\text{comod}[W_G(H)])$$

$$\simeq Ho(L_1 Sp_{G,(p)}) \simeq_\Delta Ho(Mackey_G(C^{(1,[1])}(E(1)_*E(1)-\text{comod}))).$$

Problem 1. $\mathcal{L}_1 \mathrm{Sp}_{C_5,(p)}$?

(Maybe discover better invariant than n-order).

Problem 2. $\mathrm{Sp}_{G,(p)}$ $\quad p \mid |G|$, is this rigid?

p odd.

Problem 3. $\mathrm{Ho}(KU\text{-Mod})$ is this algebraic?

$\cong$ $\qquad (|u|=2.)$

$\mathcal{D}(\mathbb{Z}[u,u^{-1}])$

triangulated? (yes, after inverting 2).

Greenlees. (1) $G = \mathbb{T} =$ circle

$MU \in \mathcal{A}(\mathbb{T})$

Q. What are its universal properties? ($/\mathbb{Q}$)

(2) $EC \rightsquigarrow C$ & functions on it.
(ellcohom)

Ex. $K \rightsquigarrow \mathbb{G}_m$ & — " —.

$\quad\ \ \|$

$(NK \longrightarrow f \otimes VK)$ $\quad VK =$ Meromorphic functions on $\mathbb{G}_m$ with poles at pts of finite order.

$= \mathbb{Q}[z,z^{-1}]\left[\frac{1}{1-z}, \frac{1}{1-z^2}, \frac{1}{1-z^3}, \dots\right].$

$t_n = \Phi_n(z)$ cyclotomic, vanishes on $\mathbb{G}_m\langle n\rangle$.

$$NK = \mathrm{Ker}\left(f \otimes VK \longrightarrow \underset{\substack{\parallel \\ \hat{H}_{\mathbb{G}_m(n)}(\mathbb{G}_m)}}{\bigoplus_n} \frac{\mathbb{Q}[z,z^{-1}]\left(\frac{1}{e_n(z)}\right)}{\mathbb{Q}[z,z^{-1}]} \right)$$

$$c^V \otimes f \longmapsto \{ \overline{t_n^{v_n} f} \}.$$

G-Spectra $\simeq d\mathcal{A}(G)$.

(2)

G	ASS	Ho	Qeq	Monoidal	E_∞^1	N_∞	E_∞^G
fin	✓	✓	✓	✓	✓	✓	✓
tori	✓	✓	✓	(done for S^1) (✓)			
O(2)	✓	✓	✓				
SO(3)	✓	✓	✓				
other							
free	✓	✓	✓		✓		
toral	✓	✓	✓				
profinite	✓	✓	✓				
parabolic							
proper discrete	✓	✓	✓				

<u>Q.</u> What about unstable models?

(connected case "Known").