ASSOCIATING CLASSES IN THE HOMOLOGY OF $\text{Aut}(F_n)$

STUDYING MODULI SPACES OF GRAPHS.

$Q$-HOMOLOGY

$\text{MG} = \text{moduli space of metric graphs (up to isometry)}$

- Graphs may have leaves, but ignore their lengths (doesn't change the $H_+$).
- No bivalent vertices.
- No units: projective classes (alternatively, $\leq \text{length of edge} = 2$).

$\text{MG}$ is specified by # loops, # leaves:

$\text{MG}_{n,m} =$ connected graphs, $T_1 \cong F_n$, $s$ leaves.

One reason to be interested:

THM (Culler-V): $n > 2$, $s = 0$, $H_+(M_{n,0}) \cong H_+(\text{Out}(F_n))$

Another reason:

THM (Kontsevich): $\text{PH}_k^h(M_{n,0}) = \text{PH}_k^{h} (sp_{\infty}) \oplus H_{n-2-k}^h (M_{n,0})$

$\text{UTC}_n$ = Lie algebra of derivations of $\text{free Lie algebra on } x_1, \ldots, x_n$:

$\text{UTC}_n$ vanishes on $\omega \in \text{E}(\mathfrak{g}_1, \mathfrak{g}_2)$
MRITA: STUDIED $\mathfrak{sl}_n$ IN CONNECTION WITH $\text{Mod}(S^3, \Sigma_5)$

**OThER CONNECTIONS TO LOW-DIM TOPOLOGY**

$\mathfrak{sp} \subset \mathfrak{l}_n$ SUBALGEBRA. ACTS ON $\mathfrak{h}_n$ BY ADJUGATE

$$\text{Pf}^*(\mathfrak{h}_n) \mathfrak{sp} = \bigoplus H_{2n-2-k}(M_{n,0})$$

**Proof of C-V:** If $X$ has $n$ loops, no loops,

THEN $\pi_0(\text{HE}(X)) \cong \text{Out}(F_n)$. Fix such an $X$,

$g : X \rightarrow G$ THEN $\equivuv$

$\text{Out}(F_n) = \pi_0(\text{HE}(X))$ ACTS ON MERGED GRAPHS.

THEN PROVE $\psi(g, G)$ IS CONVERGIBLE.

**Check:** $\text{Out}(F_n)$ Acds Properly (and

is well-defined).

If $X$ has $n$ loops and is NERVEN

THEN $\pi_0(\text{HE}(X, 2X)) = R_{n,0}$. $2X$ FIXED

POINTWISE

Fix $X = \mathfrak{g} = R_{n,0}$

MARKED GRAPHS $G \in M_{n,0}$ BY $g : X_{n,0} \rightarrow G$

$\text{Fn}_{n,0}$ ACDs. THE SPACE OF MARKED GRAPHS IS

CONVERGIBLE (HATZKE). CHECK ACTION PROPER

AND WELL-DEFINED.
\[ \Gamma_{1,2} = 2^{s-1} \times 2^{1/2} \]
\[ H_{2} \Sigma_{s} = \Lambda^{*} \otimes s+1 \]
\[ H_{+} \Gamma_{1,2} = \Lambda^{*} \otimes s+1 \]

\[ \Delta \rightarrow (F) \rightarrow \Gamma_{1,2} \rightarrow \Gamma_{1,2-n} \rightarrow 1 \]

Claim: \( H_{+} \Gamma_{1,2} \) is a \( \Sigma_{s} \) module

Assembly map

Intersection of assembly maps (with \( \Sigma_{s} \) action)

Assembly:

\[ G_{n,2i} \rightarrow G_{n,i}, \quad \Gamma_{n,2i} \rightarrow \Gamma_{n,i}, \quad H_{2}(\Gamma_{n,i}) \rightarrow H_{2}(\Gamma_{n,i}) \]

Example:

\[ \Gamma_{1,2} \times \Gamma_{1,2} \rightarrow \Gamma_{1,0} \]

\[ H_{2k}(\Gamma_{1,2k+1}) \otimes H_{2k}(\Gamma_{1,2k+1}) \rightarrow H_{2k}(\Gamma_{1,2k+1}) \]

"Braun exact sequence"
$M_3$ is the 3rd Modular Class.

$M_1, M_2, M_3$ are non-trivial in $H_k$.

Concl: (Marina) An $M_3$ non-trivial.

We computed $H^k(\Gamma_k,\mathbb{Z})$ for all $k$ as $\mathbb{Z}$-modules.

Answer: In terms of modular forms.