FOUND ACTION OF Aut(F_0) ON T(V)^\otimes^n BY ACTION OF GENERATORS. WE NEED TO VERIFY THE RELATIONS.

CAN REPLACE T(V)^\otimes^n BY H^\otimes^n FOR H CO-COMM. HOPF ALG.

HOPF AIG = VECTOR SPACE H WITH SOME MAPS:

- UNIT: \eta: K \rightarrow H
- COUNIT: \varepsilon: H \rightarrow K
- MULTIPLICATIVE: m: H \otimes H \rightarrow H
- CO-MULTIPLICATIVE: \Delta: H \rightarrow H \otimes H
- ANTIPODE: S: H \rightarrow H

\text{RELEVANT RELATIONS:} \quad +

\text{CAT\textit{EGORY}:} \quad H \text{: OBJECTS: } 0, 1, 2, \ldots

\text{MORPHISMS: GENERATED BY}

HOPF ALGEBRA = \text{FUNCTOR FROM} \quad H \quad \mathbb{Q} \rightarrow H \otimes \mathbb{Q}

\text{(SYN. MONOIDAL)}

OBSERVATION: IF H_1, H_2 ARE HOPF ALGEBRAS,

\quad y_{H_2} \rightarrow \text{Vect} \quad \text{NATURAL TRANSFORMATION BETWEEN MON. FUNCTORS} \quad HOF ALGEBRA MORPHISMS

\text{FORGET ABOUT MONOIDAL STRUCTURE.}
\text{NOT TRANSF BETWEEN FUNCTORS \rightarrow ??}
\[ \text{THM [CONVICT-K / FOLKLORE]} \]

\[ H_1 = K[F_n] \quad \text{then} \quad \text{Nat}(F_{H_1}, F_{H_2}) = \ast_{\text{Homo}} H_2 \]

More generally, for any functor,

\[ \text{Nat}(F_{H_1}, F) = F(h) \]

\[ \iff \quad \text{EQ} \quad \text{CURRENTLY,} \]

\[ \text{Hom}_G(n, m) \cong F_{n^m} \]

\[ \iff \]

\[ H_{n^m} = \text{FINITELY GENERATED FREE GROUP} \]

\[ \text{aut}(F_n) \]

\[ \text{Hom}_n = \text{Nat}(F_K[F_n], F_H) \]

\[ \text{GIVE A NEW INTERPRETATION OF THE ACTION.} \]

**FACT:** IF \( H_1 = K[G] \), \( H_2 = K[G'] \)

\[ \text{Nat}(K[G], F_K[G']) = k\text{-span}(\text{Hom}(\Gamma \to \Gamma')) \]

Now take \( H_2 = U(G) \) universal enveloping 4 LG

\[ \text{Nat}_2 \quad (K[G], F_{\text{alg}}) \quad \text{distribution \ supported \ at} \quad \text{Hom}(\Gamma \to G) \]

Such a functor is in some sense generated by one element.

But not such,

\[ \rightarrow \text{Nat}(\Gamma_{\text{alg}}, \Gamma_{\text{alg}}) \] more complicated!
**Representation Variety**

$G$ group scheme $\rightarrow k(G)$ commutative Hopf algebra

$\widehat{F}_G : \mathcal{H} \rightarrow \text{Vect}$

$F[k[r]] \otimes F_G = \bigoplus_n F(n) \otimes \hat{F}(n)$

$f \circ g = f \circ g$ in $\mathcal{H}$

Fact: $F[k[r]] \otimes F_G = k(\text{Rep}(G \rightarrow G))$

Fact: $F[k[r]] \otimes F_G = k(\text{Rep}(G \rightarrow G), \text{character variety})$

Fact: $F[k[r]] \otimes F_G = k(\text{Rep}(g \rightarrow g), \text{Lie algebra of } G)$

**Speculation:** How to quantize these constructions?

In the case of $\Gamma = \Pi_1(S)$ - surface

$\Pi_1(M)$ - 3-manifold with body

Idea: Change the category $\mathcal{H}$.

$B = \text{Prop}^{\text{op}}$ or $\mathcal{B}$ with an internals pair and morphs of framed triangles in $D \times I$.
- **Splitting** $\Rightarrow$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

- **CAN TERMINATE RIBBONS (A PAIR AT ONCE)** (COUNIT)

- **CUP AND CAPS**

- **NO KNOTS**

**FACT:** $B \xrightarrow{\text{CROSSINGs}} \mathcal{T}$

\[ \text{ie } T \text{ in } \mathbb{R}^4 \text{ instead of } \mathbb{R}^3 \]

**THEOREM:** THERE IS A **FUNCTION** $B \rightarrow \text{Vec}^+$ **DEFINED FOR ANY** + HOPF A**G** QUANTUM GROUP

**IF S IS A SURFACE**, **ANALOG** OF $\mathcal{T} \rightarrow \text{Vec}$

\[ n \rightarrow \mathbb{K}[\mathbb{T}_q(S)]^n \]

**GIVEN BY**

\[ \text{SCHAN (JANGLED IN } S \times I \text{ FROM } 0 \rightarrow n \text{ POINTS)} \]

"**CONJECTURE**": $F_S \circ B \xrightarrow{\text{FUG}} \text{UNIVERSAL QUANTUM REP}

\[ \text{Mod}(S) \text{ for the } n^\text{th} \text{ QUANTUM REP OF Mod}(S) \]