RYSZARD 4

N^1 M_c C M_c N^1 C N^1 M_0 C N^1 M_1 C

PRINCIPAL GRAPH

DUAL PRINCIPAL GRAPH

N - N BIMODULES

ROTATION \rightarrow \oplus L(M)

N + M - BIMODULES

M - N - BIMODULES

M - M - BIMODULES

PRINCIPAL GRAPH: Finite Graph by Finite Depth Assumption

FACT: Index of NCM

= Norm of Principal Graph

Operator = Biggest Eigenvalue

(= Norm of Incidence Matrix of the Graph)

COROLLARY: [M:N]

\[ \frac{1}{4} \cos^2 \frac{\pi}{n} \sum_{i=1}^{n} \left( 4_i \right) \]

even dots

\[ n \]

because norm of a \underline{MxN} matrix

with positive integer coeff
\[ M_{2+1} = \langle M_2, e_{2+1} \rangle \]

The projections \( e_1, e_2, e_3 \ldots \) satisfy that \( e_i e_j = e_j e_i \) if \(|i - j| > 1\)
and \( e_y e_3 e_y = \frac{1}{[M:N]} e_y, \quad e_3 e_y e_3 = \frac{1}{[M:N]} e_3 \)

\(\Rightarrow\) Form a temporary subalgebra.

Can also consider the relative commutant
\[ M_2 \cap M_n \subseteq M_2 \cap M_{n+1} \quad \text{more general}. \]

\[ \downarrow U \quad \downarrow U \]

\[ M_{2+1} \cap M_n \subseteq M_{2+1} \cap M_{n+1} \quad \text{more general}. \]

As diagram of finite dimensional algebra.

And the diagram of conditional expectations commute.

From such commuting squares \( A_2 \leftrightarrow A_3 \)
can construct subfactors \( A_0 \leftrightarrow A_1 \)
(constructing such squares is not easy).
Example of NAG: (G finite)

\[ C \subset C(G) \subset C(G) \times G \subset (C(G) \times G) \times G \subset \cdots \]

\[ B(l^2(G)) \quad C(G) \otimes B(l^2(G)) \]

\[ \xrightarrow{\sim} \quad C^*(G) \quad \xrightarrow{\sim} \quad B(l^2(G)) \]

\[ \xrightarrow{\sim} \quad C \quad \xrightarrow{\sim} \quad C(G) \]

\[ \text{Commuting Square} \]

\[ \text{LP with the Expectations} \]

\[ \sum (\phi_\cdot)^*(\psi_\cdot) \]

\[ \int \phi_\cdot (\psi_\cdot) \]

\[ \int \phi_{\cdot e} (\psi_\cdot) \]

\[ \int \psi_\cdot \phi_{\cdot e} \]

\[ N \subset \text{NAG} \subset \text{NAG} \times G \subset \text{NAG} \times G \times G \subset \cdots \]

\[ \#G \quad \text{N-M} \quad (\text{of the form NAG}) \]

\[ \sim \quad (\#G)^2 = \frac{\sum (\dim \pi)^2}{\pi} \]

\[ \text{Multiplicities} \]

\[ \sum \text{dim of Reps of } G \]

\[ \text{(Irreducible Reps of } G \text{)} \]
1. **N-N-Bimodule**

   \[ \Rightarrow \text{Finite Tensor Category:} \]
   \[ X \otimes Y = \oplus X_i \otimes m_i \]

   **Branding:** In general, \( \text{N-N} \neq \text{N-N} \times \text{N-N} \), but...

   **N.C.M.** \( \Rightarrow \)

   **Asymptotic Inclusion**

   Replace by

   \[ M_0 U (M_0 \cap M_0) \rightarrow M_{\infty} \]

   Also finite index

   \[ M_{\infty} = \bigcup_i M_i \]

   **Now**

   \[ \text{N-N-Bimodule} \}

   \[ \text{N-M-Bimodules} \}

   \[ \text{Braided Tensor Category} \]

   \[ [\text{Ocneanu}] \]

   **Turaev-Viro Invariant** from such categories

   **New:**

   Associate N-N-Bimodule to each \( x \cdot y \)

   and interwinners on each face

   \[ \text{by} \quad X \otimes X \rightarrow X \cdot X \]

   For \( T \in \text{End}(X) \), \( T^* \text{End}(X) \)

   \[ T \parallel_{2}^2 = \parallel (T^* T) \]

   Get 4 interwinners that can be composed \( \Rightarrow \) interwinners form Hilbert space.

   To give a self interwiner of a single bimodule \( \Rightarrow \) a number \( x \in \mathbb{C} \)

   \( \text{6j-Symbol} \)

   \[ \Rightarrow \sum \text{IT} (6j\text{-Symbol}) = \text{3-MPD Invariant} \]
Need the 

context of blindness to have 
certain properties for this to work...