André 8  

Computing the invariants

\[ S^3: \]

\[ \begin{array}{ccc}
1 & \rightarrow & 1 \\
& & \cdot \frac{1}{p}
\end{array} \]

\[ S^1 \times S^2: \]

\[ \begin{array}{ccc}
1 & \rightarrow & \frac{\Sigma}{p}
\end{array} \]

\[ \begin{array}{ccc}
\Sigma \frac{d_i}{p} & \rightarrow & \Sigma \frac{d_i}{p}
\end{array} \]

\[ \Sigma \frac{d_i}{p} = p \rightarrow 1 \]

Solid torus ("outside torus")

Solid torus ("inside torus")

\[ \Rightarrow \text{ Form } S^1 \times S^2 \text{ together.} \]

Rem.: \[ (S^2 S^2) (M) = Z (M \times S^2) \] is a 1d-TQFT

\[ (S^2 S^2) (pt) = Z (S^2) = 1 \]

\[ (S^2 S^2) (S^1 S^2) = \dim \left( \frac{Z}{S^2} (pt) \right) = 1 \]

\[ \uparrow \]

Because 1-dim TQFT

Hence we must have \[ Z(S^2 \times S^2) = 1 \] and this is why we had to have \[ \Sigma \frac{d_i}{p} = p \].
$I^3$: (Movie giving $I^3$: Turns $I^2$ submerged in the water, cover out and back in — we see the turns and its reflection)

$$\sum_{i \in P} \frac{d_i}{p} \to \sum_{i \in P} \Theta_i$$

$$\sum_{i \in P} \Theta_i$$

$$\sum_{i \in P} \frac{d_i}{p} \Theta_i$$

$S^3$ (again):

$$\sum_{i \in P} \Theta_i$$

$$\sum_{i \in P} \frac{d_i \Theta_i}{p}$$

This different computation gave a different answer than the first!

Reason: these 3-MFD invariants are in fact invariants of 3-MFD cobordant to a 4-MFD (ie. equipped with a bounding 4-MFD)
There is a 1-1 correspondence between sym. mon. functors $\text{Bord}_{1,2,3} \to \text{LinCat}$ s.t. $\mathcal{C} = \mathbb{Z}(S^2)$

(i.e. $\text{Bord}_{1,2,3} + \text{everything equipped with}$ $\text{boundary} \ 1$-mfd $\text{up to cobordism}$ $\text{for the} \ 4$-mfd $\text{is simple, and pairs } (\mathcal{C}, p)$ $\text{where } \mathcal{C}$ is a modular tensor category and $p$ is a square root of $\Sigma d_i^2$ for $d_i = d_i(\mathcal{C})$.

Def: A ribbon category is modular if the matrix

$$\begin{bmatrix}
C & \theta \times \theta \\
\theta & C
\end{bmatrix}$$

has non-zero determinant.

Cover from $\varnothing \rightarrow (0, 1) \rightarrow \varnothing$ can write it as a sequence of (non-invertible) simple monoids.

We get a matrix, which has to be invertible as the composed map is invertible.

Exercise**: $P = \sum_i d_i^2 \theta_i = p \cdot e$

Prove that!

If $p = pI$, the anomaly is trivial $\iff \text{Bord}_{1,2,3} \to \text{LinCat}$ function.
The local moves

\[ \begin{align*}
\text{Rep}^{ss}(U_q \mathfrak{sl}(2))
\end{align*} \]

<table>
<thead>
<tr>
<th>( d_i )</th>
<th>( q^6 = 1 )</th>
<th>( q^8 = 1 )</th>
<th>( q^{10} = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = \sqrt{\sum_i d_i^2} )</td>
<td>( \sqrt{2} )</td>
<td>2</td>
<td>( \sqrt{5} + 5 )</td>
</tr>
<tr>
<td>( \frac{1}{2\pi} \text{ arg}(\Theta_i) )</td>
<td>0, ( \frac{1}{4} )</td>
<td>0, ( \frac{3}{16}, \frac{1}{2} )</td>
<td>0, ( \frac{3}{20}, \frac{2}{5}, \frac{3}{4} )</td>
</tr>
<tr>
<td>( p_t = \sum_i d_i^2 \Theta_i )</td>
<td>1+i</td>
<td>2 ( e^{\frac{2\pi i}{16}} )</td>
<td>(( \sqrt{5} + 5 )), ( e^{\frac{2\pi i}{40}} )</td>
</tr>
</tbody>
</table>

"5L(2) level k": \( q^{2(k+2)} = 1 \), \( d_i = [i+1]_q \); \( \Theta_i = q^{\frac{1}{2}i^2+i} \); \( p_t = p \cdot e^{\frac{2\pi i}{8(k+2)}} \)