



Steven Deprez

september 15, 2011



construction

applications

•

Type II₁ factors

Definition

A von Neumann algebra M is a (separable) type II₁ factor if

- *M* has trivial centre: $\mathcal{Z}(M) = \mathbb{C}$
- *M* has a trace $\tau: M \to \mathbb{C}$
 - au is a state
 - $\tau(xy) = \tau(yx)$
 - faithful, normal
- ▶ we require that *M* acts on a separable hilbert space

Examples

- Group von Neumann algebra $L(\Gamma)$ of an ICC group
- $L^{\infty}(X) \rtimes \Gamma$ of a free, p.m.p. ergodic action

Question

When do different constructions give different II₁ factors?

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The fundamental group of a II_1 factor

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History Type II₁ factors

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Type II_1 factors

History

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A von Neumann algebra M is a type II₁ factor if

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Examples

Group von Neumann algebra L(Γ) of an ICC group
 L[∞](X) × Γ of a free, p.m.p. ergodic action

Question

When do different constructions give different II₁ factors?



applications

Type II_1 factors

History

Definition

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Question

When do different constructions give different II_1 factors?



► $R \neq L(\mathbb{F}_n)$ for any $1 < n \le \infty$ (Murray-von Neumann, 1943) ► $|s| (\mathbb{F}_n) = L(\mathbb{F}_n)$ if $n \neq m^2$

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▶ $R \neq L(\mathbb{F}_n)$ for any $1 < n \leq$ ▶ Is $L(\mathbb{F}_n) = L(\mathbb{F}_m)$ if $n \neq m$?

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(Major open problem)



► Is $L(\mathbb{F}_n) = L(\mathbb{F}_m)$ if $n \neq m$?

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The fundamental group of a II₁ factor

(Major open problem)

construction



Definition and history



- ▶ Let *M* be a II₁ factor
 - ▶ if $p \in M$ is a projection, then pMp is again a II₁ factor
 - up to isomorphism, it only depends on $\tau(p)$
- The fundamental group is $\mathcal{F}(M) = \{\tau(p)/\tau(q) \mid pMp \cong qMq\} \subset \mathbb{R}_+^{\times}.$ • this is a subgroup of \mathbb{R}^{\times}
- ▶ this is a subgroup of ℝ[×]₊

Examples



applications

Definition (Murray-von Neumann, 1943)

• Let M be a II₁ factor

Definition and history

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 F(M) = {τ(p)/τ(q) | pMp ≅ qMq} ⊂ ℝ[×]₊.

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Examples

$$\mathcal{F}(R) = \mathbb{R}_{+}^{\times}$$

$$\mathcal{F}(L(\mathbb{F}_{\infty})) = \mathbb{R}_{+}^{\times}$$

$$\mathcal{F}(L(\mathbb{F}_{n})) =? \text{ for } 1 < n < \infty$$

construction

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$$\mathcal{F}(M) = \{ au(p)/ au(q) \mid pMp \cong qMq\} \subset \mathbb{R}_+^{ imes}.$$

• this is a subgroup of \mathbb{R}_+^{\times}

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▶
$$\mathcal{F}(R) = \mathbb{R}^{\times}_{+}$$
 (Murray-von Neumann, 1943)
▶ $\mathcal{F}(L(\mathbb{F}_{\infty})) = \mathbb{R}^{\times}_{+}$ (Radulescu, 1992)
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construction

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Definition and history

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$$\mathcal{F}(M) = \{ \tau(p) / \tau(q) \mid pMp \cong qMq \} \subset \mathbb{R}_+^{\times}.$$

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Definition and history

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equivalent to isomorphism problem

(Radulescu, 1992)

construction

applications

Definition and history



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 for $1 < n < \infty$



applications



Definition and history

$$\mathcal{F}(M) = \{ au(p) / au(q) \mid pMp \cong qMq\} \subset \mathbb{R}_+^{ imes}$$
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Question (Murray-von Neumann, 1943)

Which subgroups of \mathbb{R}_+^{\times} are fundamental groups?

- only \mathbb{R}^{\times}_{+} itself?
- only \mathbb{R}^{\times}_+ and countable?

- ► $\mathcal{F}(L(\Gamma))$ is countable if Γ has ICC, (T) (Connes, 1980) ► $\mathcal{F}(L(SL_2 \mathbb{Z} \ltimes \mathbb{Z}^2)) = \{1\}$ (Popa, 2002)
- $\mathcal{F}(M)$ can be any countable subgroup of $\mathbb{R}_+^{ imes}$ (Popa, 2003)
- ▶ many uncountable groups are $\mathcal{F}(M)$ (Popa–Vaes, 2008)
- My result

construction

applications

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applications





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Which subgroups of \mathbb{R}^{\times}_+ are fundamental groups?

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- $\mathcal{F}(M)$ can be any countable subgroup of \mathbb{R}^{\times}_+ (Popa, 2003)
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Definition and history



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- ► $\mathcal{F}(L(\Gamma))$ is countable if Γ has ICC, (T) (Connes, 1980)
 - we can not compute $\mathcal{F}(L(\Gamma))$ for any ICC property (T) group

•
$$\mathcal{F}(\mathsf{L}(\mathsf{SL}_2 \mathbb{Z} \ltimes \mathbb{Z}^2)) = \{1\}$$

- ▶ $\mathcal{F}(M)$ can be any countable subgroup of \mathbb{R}_+^{\times} (Popa, 2003)
- many uncountable groups are $\mathcal{F}(M)$
- ► My result

applications

Definition and history



Question (Murray-von Neumann, 1943)

Which subgroups of \mathbb{R}^{\times}_{+} are fundamental groups?

- only \mathbb{R}^{\times}_+ itself? No
- only $\mathbb{R}^{\times}_{\perp}$ and countable?

- \blacktriangleright $\mathcal{F}(L(\Gamma))$ is countable if Γ has ICC, (T) (Connes, 1980) (Popa, 2002)
- $\blacktriangleright \mathcal{F}(\mathsf{L}(\mathsf{SL}_2 \mathbb{Z} \ltimes \mathbb{Z}^2)) = \{1\}$
 - deformation/rigidity
- $\blacktriangleright \mathcal{F}(M)$ can be any countable subgroup of $\mathbb{R}^{\times}_{\perp}$ (Popa, 2003)
- many uncountable groups are $\mathcal{F}(M)$ (Popa–Vaes, 2008)
- ▶ My result

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- $\mathcal{F}(M)$ can be any countable subgroup of \mathbb{R}_+^{\times} (Popa, 2003)
 - ► alternative constructions: Ioana-Peterson-Popa, Houdayer
- ▶ many uncountable groups are $\mathcal{F}(M)$ (Popa–Vaes, 2008)
- ► My result

applications



Definition and history

$$\mathcal{F}(M) = \{ au(p) / au(q) \mid pMp \cong qMq\} \subset \mathbb{R}^{ imes}_+$$
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Question (Murray–von Neumann, 1943)

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Definition and history

$$\mathcal{F}(M) = \{ \tau(p) / \tau(q) \mid pMp \cong qMq \} \subset \mathbb{R}_+^{\times}.$$

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- many uncountable groups are $\mathcal{F}(M)$
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(Popa–Vaes, 2008)

applications



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- many uncountable groups are $\mathcal{F}(M)$
 - pure existence result
- My result

(D., 2010)

(Popa-Vaes, 2008)

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- ▶ many uncountable groups are $\mathcal{F}(M)$ (Popa–Vaes, 2008)
- My result
 - explicit construction
 - potentially larger class of groups

(D., 2010)

construction

applications



Definition and history

$\mathcal{F}(M) = \{ \tau(p) / \tau(q) \mid pMp \cong qMq \} \subset \mathbb{R}_+^{\times}.$

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(D., 2010)

construction

applications



The construction

- \blacktriangleright We begin with two actions and a quotient
 - "generic action": $\Lambda \curvearrowright (Y, \nu)$: ergodic, inf. m. p.
 - "specific action": $\Gamma \curvearrowright (X, \mu)$: free, ergodic, p.m.p.
 - quotient $\pi : \Gamma \to \Lambda$.

• $\Gamma \frown X \times Y$ by $g(x, y) = (gx, \pi(g)y)$

- ► free, i.m.p.
- ergodic if ker $\pi \curvearrowright(X,\mu)$ is ergodic
- $M = L^{\infty}(X \times Y) \rtimes \Gamma$: a II_{∞} factor
 - every isomorphism $\psi: M \to M$ scales Tr by $mod(\psi)$.
 - if $Tr(p) < \infty$, then pMp is a II₁ factor.
 - $\mathcal{F}(pMp) = mod(Aut(M))$
- $mod(Centr_{Aut(Y,\nu)}(\Lambda)) \subset mod(Aut(M))$
 - if $\Delta \in \text{Centr}_{Aut(Y,\nu)}(\Lambda)$, then id $\times \Delta$ commutes with Γ
 - ▶ so $\psi(au_g) = (\operatorname{id} \times \Delta)_*(a)u_g$ defines an automorphism of M
- ▶ Strong conditions on $\Gamma \curvearrowright (X, \mu)$: mod(Centr_{Aut(Y,\nu)}(Λ)) = mod(Aut(M))
 - Popa–Vaes conditions: no explicit examples + Λ amenable + $\Lambda \curvearrowright Y$ free
 - my set of conditions: explicit examples + all Λ + non-free actions



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• $\Gamma \frown X \times Y$ by $g(x, y) = (gx, \pi(g)y)$

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• $M = L^{\infty}(X \times Y) \rtimes \Gamma$: a II_{∞} factor

- every isomorphism $\psi: M \to M$ scales Tr by $mod(\psi)$.
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Example

Examples

History

There are examples with $\pi: \Gamma \to \Lambda = \mathbb{F}_{\infty}$.

Corollary

For any ergodic, i.m.p. $\alpha : \Lambda \curvearrowright (Y, \nu)$ of **any** group, there is a II₁ factor M_{α} with

$$\mathcal{F}(M_{\alpha}) = \mathsf{mod}(\mathsf{Centr}_{\mathsf{Aut}(Y,\nu)}(\Lambda))$$

Corollary

For any closed subgroup $\mathcal{G} \subset \operatorname{Aut}_{\nu}(Y)$ that acts ergodically on Y, there is a type II₁ factor $M_{\mathcal{G}}$ such that $\mathcal{F}(M_{\mathcal{G}}) = \operatorname{mod}(\operatorname{Centr}_{\operatorname{Aut}(Y,\nu)}(\mathcal{G})).$

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History

• We replace (Y, ν) by a II_{∞} factor (B, Tr).

- We replace $\Lambda \curvearrowright (Y, \nu)$ by an outer action $\alpha : \Lambda \to \operatorname{Out}_{\mathsf{Tr}}(B)$.
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Corollary

- Gives an alternative characterization of all fundamental groups
- in terms of outer actions on abritrary II_{∞} factors *B*: harder
- conjecture: we can assume that $B = L(\mathbb{F}_{\infty})^{\infty}$

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