Bachelor projects for mathematics and mathematics-economics

Department of Mathematical Sciences
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**Introduction**

This is a catalogue of projects suggested by the researchers at the Department of Mathematical Sciences for students in the B.S. programs in mathematics and mathematics-economics. It is important to note that such a catalogue will never exhaust all possibilities – indeed, if you are not finding what you are looking for you are strongly encouraged to ask the member of our staff you think is best qualified to help you on your way for suggestions of how to complement what this catalogue contains. Also, the mathematics-economics students are encouraged to study the searchable list of potential advisors at the Economy Department on


If you do not know what person to approach at the Department of Mathematical Sciences, you are welcome to try to ask

- the director of studies (Ernst Hansen, erhansen@math.ku.dk) or
- the associate chair for education (Jesper Lützen, lutzen@math.ku.dk).

When you have found an advisor and agreed on a project, you must produce a contract (your advisor will know how this is done), which must then be approved by the director of studies at the latest during the first week of a block. The project must be handed in during the 7th week of the following block, and an oral defense will take place during the ninth week.

We wish you a successful and engaging project period!

Best regards,

Jesper Lützen  
Associate chair

Ernst Hansen  
Director of studies
1 Finance

1.1 Rolf Poulsen
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Relevant interests:

*Finance.*

Suggested projects:

- **Option pricing [Fin1]**

- **Stochastic interest rates [Fin1]**

- **Optimal portfolio choice [Fin1]**
  The effects of parameter uncertainty on optimal portfolio choice (Kan & Zhou (2007) for instance). *Betting Against Beta* (Frazzini & Pedersen (2014)) and other CAPM-related stuff. Optimal multi-period investment with return predictability and transaction costs (Garleanu & Pedersen (2013)).

- **Stochastic volatility [Fin1 and preferably, but not necessarily, FinKont]**
  What is volatility? (There are at least a handful of definitions that all more or less are the same in the Black-Scholes model – but not in more general models.) How does volatility behave empirically? How does (stochastic) volatility affect (plain vanilla) option pricing? How can variance swaps (and other volatility derivatives) be priced and hedged?

1.2 Other projects

Other projects in this area can be found with

- Jens Hugger (4.2)
2 Operations research

2.1 Trine K. Boomsma and Salvador Pineda

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Relevant interests:

Mathematical programming, stochastic programming, dynamic programming, real options, energy applications.

Suggested projects:

- **Multi-stage Stochastic Programming vs. Stochastic Dynamic Programming** [OR1 + one of the following courses: Advanced mathematical programming, OK, GAMS, OR2, Stochastic Programming]

Stochastic programming is a mathematical framework that allows to solve optimization problems when some of the parameters involved are not fully known. For example, stochastic programming can be used to address both everyday issues such as “which road should I take to get to work without knowing the traffic level in each of them?” and complicated problems such as determining the optimal portfolio of assets in a market without knowing how prices will evolve.

However, the decisions made in the two previous examples are not final, but can be modified as uncertain information discloses. For example, if we decide to drive to work through road A and we realize after some kilometres that the traffic is very high, we may be able to switch to road B. Likewise, if we observe high prices of a given asset, we will try to increase the share of such asset to improve our final profit. Multi-stage stochastic programming succeeds to solve problems in which decisions may change as uncertain information becomes available. However, these type of problems present the curse of dimensionality, which basically means that the number of equations and variables drastically increase with the number of decision stages.

Alternatively, these problems can be solved using dynamic programming, which is a methodology that requires that decisions at each stage are discretized. For example, in the case of the assets, we should impose that assets can be sold or bought in lots of 10. By doing so, we can obtain the solution to the initial problem by solving smaller subproblems. While dynamic programming reduces the computational burden of some models, it does not provide the exact optimal solution because of the discretization of the decision variables.

Within this framework, this project will focus on comparing these two mathematical tools and determine in which cases multi-stage stochastic programming overcomes dynamic programming and vice versa. To evaluate each methodology in a real-life problem, we propose an energy planning model that aims at determining the electricity production of different types of generating units facing the uncertainty of electricity demand level and wind power production.
3 Algebra and number theory

3.1 Henrik Holm
holm@math.ku.dk

Relevant interests:
Rings, modules, homological algebra, category theory.
The prerequisites for the following projects are the courses [Alg 1] and [Alg 2].
Details, and possibly additional suggestions for projects, may be found at my homepage http://www.math.ku.dk/~holm/

Suggested projects:

- **Completion of rings [Alg1, Alg2]**
  Given an ideal \( I \) in a commutative ring \( R \) one can construct the \( I \)-adic completion \( \hat{R}_I \). For example, \( \hat{k}[x]_{(x)} \) is the formal power series ring and \( \hat{\mathbb{Z}}_{(p)} \) is the ring of \( p \)-adic integers. The aim of this project is to define adic completions and to investigate their basic properties.
  Literature: H. Matsumura, “Commutative ring theory”.

- **Group (co)homology [Alg1, Alg2]**
  To a group \( G \) one can associate a sequence of (abelian) homology groups \( H_n(G) \) and cohomology groups \( H^n(G) \) that contain information about \( G \). For example, \( H_1(G) = G_{ab} \) is the abelianization of \( G \). The aim of this project is to define group (co)homology and to give group theoretical descriptions of the lower (co)homology groups.
  Literature: P. J. Hilton and U. Stammbach, “A course in homological algebra”.

- **Gröbner bases [Alg1, Alg2]**
  Given an ideal \( I \) in the polynomial ring \( k[x_1, \ldots, x_n] \) and a term ordering \( \leq \) one can always find a so-called Gröbner basis \( g_1, \ldots, g_m \) of \( I \) with respect to \( \leq \). For example, a Gröbner basis for the ideal \( I = (y^2 - x^3 + x, y^3 - x^2) \) with respect to the lexicographic ordering (where \( x \geq y \)) consists of \( g_1 = y^2 - 2y - 2y^3 - y^4 \) and \( g_2 = x - y^2 + y^3 + y^4 \). Gröbner bases are powerful tools to solve e.g. polynomial equations and the ideal membership problem. The aim of this project is to define, and to show the existence of, Gröbner bases, and to demonstrate some applications.
  Literature: N. Lauritzen, "Concrete abstract algebra" and D. Cox, J. Little, and D. O'Shea, “Ideals, varieties, and algorithms”.

- **Injective modules [Alg1, Alg2]**
  An object in a category is called injective if it has a certain lifting property. For example, the injective objects in the category of abelian groups are precisely the divisible abelian groups (such as the group of rational numbers \( \mathbb{Q} \) and the Prüfer groups \( \mathbb{Z}(p^\infty) \) where \( p \) is a prime). The aim of this project is to develop the theory of injective modules over an arbitrary ring.
The algebraic K-theory of a ring $R$ is a certain sequence $K_n(R)$ of abelian groups that contains information about $R$. For example, if $R$ is a field, then $K_0(R) = \mathbb{Z}$ is the additive group of integers and $K_1(R) = R^*$ is the multiplicative group of units in $R$. The aim of this project is to define and investigate the lower K-groups for certain classes of rings.

Adjoint functors are important and abundant in mathematics. For example, the forgetful functor $U: \text{Vct} \to \text{Set}$ from the category of (real) vector spaces to the category of sets has a left adjoint $V: \text{Set} \to \text{Vct}$, which to each set $X$ associates the (real) vector space with basis $X$. The aim of this project is to develop the basic theory of adjoint functors and to prove Freyd’s Adjoint Functor Theorem and The Special Adjoint Functor Theorem.

### 3.2 Christian U. Jensen

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Relevant interests:

* Galois theory. Algebraic number theory.*

Suggested projects:

- **Introductory Galois theory** [Alg2]
  
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics, but is also a very rich field that can be studied in its own right.

- **Introduction to algebraic number theory** [Alg2]
  
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

### 3.3 Ian Kiming

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Relevant interests:

*Algebraic number theory and arithmetic geometry.*

Suggested projects:

- **Introduction to algebraic number theory [Alg2]**
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

- **First case of Fermat’s last theorem for regular exponents [Alg2]**
  The project studies the proof of Fermat’s last theorem for ‘regular’ prime exponents $p$ in the so-called first case: this is the statement that $x^p + y^p + z^p = 0$ does not have any solutions in integers $x, y, z$ not divisible by $p$. The project involves studying some introductory algebraic number theory which will then also reveal the definition of ‘regular primes’.

- **p-adic numbers [Alg2]**
  The real numbers arise from the rational numbers by a process called ‘completion’. It turns out that the rational numbers (and more generally any algebraic number field) has infinitely many other ‘completions’, namely one associated to each prime number $p$. The fields that arise in this way are called the fields of $p$-adic numbers. They have a lot of applications in many branches of mathematics, not least in the theory of Diophantine equations, i.e., the question of solving in integers polynomial equations with integral coefficients.

- **Hasse–Minkowski’s theorem on rational quadratic forms [Alg2]**
  A rational quadratic form is a homogeneous polynomial with rational coefficients. The Hasse–Minkowski theorem states that such a polynomial has a non-trivial rational zero if and only if it has a non-trivial zero in the real numbers and in all fields of $p$-adic numbers. The latter condition can be translated into a finite number of congruence conditions modulo certain prime powers and thus one obtains an effective criterion. The project involves an initial study of $p$-adic numbers.

- **Continued fractions and Pell’s equation [Alg2]**
  The project studies the theory of continued fractions and how this can be applied to determining units in quadratic number rings. This has applications to the study of Pell (and ‘non-Pell’) equations, i.e., solving equations $x^2 - Dy^2 = \pm 1$ in integers for a given positive, squarefree integer $D$.

- **Class groups of quadratic number fields and binary quadratic forms [Alg2]**
A quadratic number field is a field obtained from $\mathbb{Q}$ by adjoining a number of the form $\sqrt{D}$ where $D$ is an integer that is not a square (in $\mathbb{Z}$.) The class group attached to such a field measures how far its so-called ring of integers in from being a unique factorization domain. These class groups are necessary to study if one wants to understand integer solutions to equations of form $ax^2 + by^2 = c$ for given integers $a, b, c$.

- **Modular forms on SL$_2(\mathbb{Z})$** [Alg2, KomAn]
  This project studies modular forms on SL$_2(\mathbb{Z})$. These are initially analytic objects and thus a certain, minimal background in complex analysis is required. Modular forms turn out to have a lot of deep connections to arithmetic, and one can use this project as a platform for a later study of the more general modular forms on congruence subgroups of SL$_2(\mathbb{Z})$. These are very important in modern number theory and are for instance central in Andrew Wiles’ proof of Fermat’s last theorem.

- **Introductory Galois theory** [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics.

- **Group cohomology** [Alg2]
  Group cohomology is a basic and enormously important mathematical theory with applications in algebra, topology, and number theory. The project will study the initial theory staring with cohomology of discrete groups and then perhaps move on to cohomology of profinite groups. This project can be used as a platform for continuing with study of Galois cohomology and Selmer groups.

- **The theorem of Billing–Mahler** [Alg2, EllKurv]
  A big theorem of Barry Mazur (1977) implies in particular that if $n$ is the order of a rational point of finite order on an elliptic curve defined over $\mathbb{Q}$ then either $1 \leq n \leq 10$ or $n = 12$. Thus, in particular, $n = 11$ is impossible. This latter statement is the theorem of Billing and Mahler (1940). The project studies the proof of the theorem of Billing–Mahler which will involve a bit more theory of elliptic curves as well as an initial study of algebraic number theory. The impossibility of $n = 13$ can also be proved with these methods.

- **Torsion points on elliptic curves** [Alg2, EllKurv]
  The project continues the study of elliptic curves defined over $\mathbb{Q}$ in the direction of a deeper study of (rational) torsion points. There are several possibilities here, for instance, parametrizations of curves with a point of a given, low order, generalizations of the Nagell–Lutz theorem, the structure of the group of torsion points on elliptic curves defined over a $p$-adic field (Lutz’ theorem).

- **Primality testing** [Alg2]
  How can one decide efficiently whether a large number is a prime number? The project will study one or more of the mathematically sophisticated methods of...
doing this: the Miller–Rabin probabilistic primality test and/or the more recent Agrawal-Kayak-Saxena deterministic primality test. The project will include an initial study of algorithmic complexity theory.

- **Factorization algorithms** [Alg2]
  How can one find the prime factorization of a large number? The project will study one or more of the mathematically sophisticated methods of doing this: the Dixon factorization method, factorization via continued fractions, the quadratic sieve. The project will include an initial study of algorithmic complexity theory.

- **Open project** [?]
  If you have some ideas on your own for a project within the general area of number theory, you can always come and discuss the possibilities with me.

**Previous projects:**

- **The Agrawal-Kayak-Saxena primality test** [Alg2]
- **Selmer groups and Mordell’s theorem** [Alg3, EllKurv]
- **Hasse–Minkowski's theorem on rational quadratic forms** [Alg2]
- **Torsion points on elliptic curves** [Alg2, EllKurv]
- **Factorization via continued fractions** [Alg2, Krypto]
- **The Pohlig-Hellman algorithm for computing discrete logarithms** [Alg2]
- **Schoof’s algorithm** [Alg3, EllKurv]

**3.4 Other projects**

Other projects in this area can be found with

- Christian Berg (??)
- Jesper Grodal (7.1)
- Morten S. Risager (4.4)
4 Analysis

4.1 Bergfinnur Durhuus
durhuus@math.ku.dk

Relevant interests:

Suggested projects:

- **Graph colouring problems** [Dis1, An1]
  Problems originating from various areas of mathematics can frequently be formulated as colouring problems for certain types of graphs. The four-colour problem is probably the best known of colouring problems but there is a variety of other interesting colouring problems to attack

- **Combinatorics of graphs** [Dis1, An1, ComAn]
  Counting of graphs specified by certain properties (e.g. trees) is one of the classical combinatorial problems in graph theory having applications in e.g. complexity theory. The method of generating functions is a particularly effective method for a large class of such problems making use of basic results from complex analysis

- **Unbounded operators and self-adjointness** [An2]
  Many of the interesting operators playing a role in mathematical physics, in particular differential operators of use in classical and quantum mechanics, are unbounded. The extension of fundamental results valid for bounded operators on a Hilbert space, such as the notion of adjoint operator and diagonalisation properties, is therefore of importance and turns out to be non-trivial

Previous projects:

- **Clifford algebras, Spin groups and Dirac operators** [Alg1, An2]
- **Ramsey theory** [Dis1, An1]
- **Causal Structures** [An1, Geom2]
- **The Tutte polynomial** [Dis1, An1]
- **Knot theory and statistical mechanics** [Dis1, AN1]
- **Graph 3-colourings** [Dis1, An1]
- **Minimal surfaces** [Geom1, An1]
- **Planar graphs** [Dis1, AN1]
4.2 Jens Hugger
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Relevant interests:
* Numerical analysis – eScience

Suggested projects:

- **The Gamma function** [An1,KomAn]
  Euler’s Gamma function is the most important of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.

- **Entire functions** [An1, Koman]
  Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.

- **Numerical methods for pricing options** [NumIntro, preferably also NumDiff]
  Pick one or more options and “solve” it with one or more numerical methods. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the option, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- **Numerical methods for optimization** [NumIntro, preferably also NumDiff]
  Pick an optimization problem and “solve” it with one or more numerical methods. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the optimization problem, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- **Numerical methods for differential equations** [NumIntro, NumDiff]
  Pick a differential equation and solve it with a numerical method. Either bring your own problem or get one from the advisor. The project can focus on different aspects like mathematical modelling of the problem, programming and comparison of numerical methods for the solution, theory of convergence and stability for the numerical methods.

- **Convergence of numerical methods for PDE’s** [An2]
  Learn the theory of convergence analysis for numerical methods for PDE’s. Apply the theory to a real life problem (of your choice or provided by me like for example the Asian option from finance theory.)
Numerical methods for interpolation or integration in several dimensions or iterative solution of large equation systems [NumIntro]
Pick a problem and solve it with a numerical method. Either bring your own problem or get one from the advisor.

Porting part of a Maple program into a fast programming language [NumIntro, Programming experience with Maple and a fast programming language like Fortran, C etc]
Replace the slow part of a Maple code for solving an Asian option with code written in a faster language. Document your code.

Previous projects:

- Convection-diffusion in one variable [NumDiff]
- Asian options [NumDiff]

4.3 Henrik L. Pedersen
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Relevant interests:
Complex analysis. Special functions. Orthogonal polynomials and moment problems.

Suggested projects:

- The Gamma function [An1,KomAn]
  Euler’s Gamma function is the most import of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.

- Entire functions [An1, Koman]
  Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.

- Boundary behaviour of power series [KomAn]
  A power series converges inside the disk of convergence, and diverges outside the circle. What happens on the boundary? The sum of the geometric series has a holomorphic extension to the entire complex plane except 1. If we remove a lot of terms from this series it turns out that the sum function has the unit circle as a natural boundary, meaning that it cannot be extended holomorphically to any arc of that circle. What is going on?

- Subharmonic functions [KomAn, Measure theory]
  The real part $u$ of a holomorphic function is harmonic, meaning that its Laplacian is zero: $\Delta u = \partial^2_{xx} u + \partial^2_{yy} u \equiv 0$. A subharmonic function $u$ in the complex
plane satisfies $\Delta u \geq 0$. For these functions versions of the maximum principle and of Liouville’s theorem hold

- **PICARD’S THEOREMS** [KomAn, some measure theory]
  If you are presented with an entire function $f$ and you have two different complex numbers not in the image set $f(\mathbb{C})$ then $f$ is constant. This result is known as Picard’s little theorem.

- **RIEMANN’S MAPPING THEOREM** [KomAn]
  Any simply connected region $\mathcal{D}$ in the complex plane except the plane itself is conformally equivalent to the open unit disk $\Delta$, meaning that there exists a holomorphic and bijective mapping $\varphi : \Delta \rightarrow \mathcal{D}$.

- **MÜNTZ-Szasz’ THEOREM** [KomAn, Functional Analysis]
  Let $\{\lambda_k\}$ be an increasing sequence of positive numbers. When is the span of the power-functions $\{1, x^{\lambda_1}, x^{\lambda_2}, \ldots\}$ dense in the space of continuous functions on $[0, 1]$? Answer: exactly when $\sum_{k=1}^{\infty} 1/\lambda_k = \infty$!

- **PALEY–WIENER’S THEOREM** [KomAn, some measure theory]
  The Fourier transform $\hat{\varphi}$ of a function $\varphi$ from the Hilbert space $L_2(-a,a)$ can be extended to an entire function of exponential type (meaning that its growth is dominated by $e^{K|z|}$ for all large $|z|$). Conversely, any entire function of exponential type is in fact the Fourier transform of an $L_2$-function of a finite interval.

4.4 Morten S. Risager

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**Relevant interests:**

*Number theory, automorphic forms, complex analysis, Riemann surfaces.*

**Suggested projects:**

- **THE PRIME NUMBER THEOREM** [KomAn, An2]
  The prime number theorem gives a quantitative version of Euclid theorem about the infinitude of primes: it describes how the primes are distributed among the integers. It was conjectured 100 years before the first proof.

- **TWIN PRIMES AND SIEVE THEOREMS** [KomAn, An2]
  Very little is known about the number of twin primes. Using sieve methods one can show that the sum of reciprocals of twin primes is convergent. Still it is not known if there are only finitely many or not.

- **THE FUNCTIONAL EQUATION FOR RIEMANN’S ZETA FUNCTION** [KomAn, An2]
  Using methods from Fourier analysis - in particular Poisson summation - one investigates the properties of Riemann’s famous zeta function.
Counting elements in free groups \([\text{KomAn, An2}]\)

How does one count in a reasonable way the number of elements in the free group on \(n\) generators? Using methods from linear algebra one can give good asymptotic and statistical results. Numerical investigations is also a possibility.

Previous projects:

- **Elementary methods in number theory, and a theorem of Terrence Tao.** [An2, ElmTal]
- **Primes in arithmetic progressions** [KomAn, An2]
- **Small eigenvalues of the automorphic Laplacian and Rademachers conjecture for congruence groups** [KomAn, An3]

4.5 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- **The Heisenberg group** [An1, An2]
  The Heisenberg group is important, for example because it is generated by the position and momentum operators in quantum mechanics. The purpose of this project is to study its representation theory. A famous theorem of Stone and von Neumann relates all irreducible representations to the Schrödinger representation acting on \(L^2(\mathbb{R}^n)\).

- **Uncertainty principles** [An1, Sand1, KomAn]
  Various mathematical formulations of the Heisenberg uncertainty principle are studied. Expressed mathematically, the principle asserts that a non-zero function \(f\) on \(\mathbb{R}\) and its Fourier transform \(\hat{f}\) cannot be simultaneously concentrated. A precise version, called the Heisenberg inequality, expresses this in terms of standard deviations. A variant of the theorem, due to Hardy, states that \(f\) and \(\hat{f}\) cannot both decay more rapidly than a Gaussian function.

- **The Peter-Weyl theorem** [An1, An2, Sand1]
  The purpose of this project is to study \(L^2(G)\) for a compact group \(G\), equipped with Haar measure. The theorem of Peter and Weyl describes how this space can be orthogonally decomposed into finite dimensional subspaces, which are invariant under left and right displacements by \(G\). Existence of Haar measure can be proved or assumed.
5 Geometry

5.1 Henrik Schlichtkrull
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Relevant interests:
Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- **Global properties of curves (and/or surfaces)** [Geom1,An1]
The differential geometry studied in Geometry 1 is of a local nature. The curvature of a curve in a point, for example, describes a property of the curve just in the vicinity of that point. In this project the focus is on global aspects of closed curves, as for example expressed in Fenchel’s theorem, which gives a lower bound for the total integral of the curvature, in terms of the perimeter.

- **Geodesic distance** [Geom1,An1]
The geodesic distance between two points on a surface is the shortest length of a geodesic joining them. It turns the surface into a metric space. The project consists of describing some properties of the metric. For example Bonnet’s theorem: *If the Gaussian curvature is everywhere $\geq 1$, then all distances are $\leq \pi$.*

5.2 Hans Plesner Jakobsen
jakobsen@math.ku.dk

Relevant interests:
Unitaritet, Liegrupper, Liealgebraer, kvantiserede matrixalgebraer, kovariante differentialoperatorer i matematisk fysik, kvantiserede indhyldningsalgebraer

Suggested projects:

- **Symmetrier** [ca. 1 års matematik]
  - Diskrete symmetrier: Tapetgrupper, Krystallografiske grupper.
  - Kontinuerne symmetrier: Rotationsgruppen, Lorentzgruppen, Poincaré-gruppen, den konforme gruppe, ...

- **Gauss-Bonnet** [Forudsætter ca. 1 års matematik]
• **DE KANONISKE KOMMUTATORRELATIONER** [ca. 1 års matematik + Hilbertrum]
  Operatorerne $Q$ og $P$ givet ved $(QF)(x) = xF(x)$ og $(PF)(x) = -i\frac{dF}{dx}(x)$ er forbundet via Fouriertransformationen, men kan Fouriertransformationen 'konstrueres' ud fra disse? Kan man bygge en bølgeoperator eller en Diracoperator ud af den harmoniske oscillator?

• **LIEALGEBRAER** [ca. 1 års matematik + (kan aftales)]
  (Eks.) Klassifikation. Dynkin diagrammer, Kac-Moody algebraer, super Liealgebraer. Hvad fik Borcherds (bl.a.) Fieldsmedaljen for?

• **MATRIX LIEGRUPPER** [ca. 1 års matematik]
  Bl.a. eksponentielfunktionen for matricer, tensorprodukter, duale vektorrum. Er der en forbindelse mellem Peter Weyl Sætning og Stone Weirstrass Sætning?

### 5.3 Other projects

Other projects in this area can be found with

- Ib Madsen (7.2)
- Nathalie Wahl (7.4)
6 Noncommutativity

6.1 Søren Eilers
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Relevant interests:

Advanced linear algebra related to operator algebras. Dynamical systems. Mathematics in computer science; computer science in mathematics.

Suggested projects:

- **PERRON-FROBENIUS THEORY WITH APPLICATIONS** [LinAlg, An1]
  Methods involving matrix algebra lead to applications such as Google's PageRank and to the ranking of American football teams.

- **DATA STORAGE WITH SYMBOLIC DYNAMICS** [An1, Dis1]
  Engineering constraints necessitate a recoding of arbitrary binary sequences into sequences meeting certain constraints such as “between two consecutive ones are at least 1, and at most 3, zeroes”. Understanding how this is done requires a combination of analysis and discrete mathematics involving notions such as entropy and encoder graphs.

- **EXPERIMENTAL MATHEMATICS** [LinAlg]

Previous projects:

- **AN EXPERIMENTAL APPROACH TO FLOW EQUIVALENCE** [An1]
- **VISUALIZATION OF NON-EUCLIDEAN GEOMETRY** [MatM, Geom1]
- **PLANAR GEOMETRY IN HIGH SCHOOL MATHEMATICS** [MatM]
- **LIAPOUNOV’S THEOREM** [MI]

6.2 Niels Grønbæk
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Relevant interests:

Banachrum, banachalgebra, kohomologi, matematikkens didaktik
Suggested projects:

- **Et undervisningsforløb på gymnasialt niveau** [LinAlg, An1, Alg1, Geo1]
  Projektet går ud på at tilrettelægge, udføre og evaluere et undervisningsforløb af ca. 2 ugers varighed i en gymnasieklasse.

Suggested projects:

- **Amenable Banach Algebras** [An3]
  Amenability of Banach algebras is an important concept which originates in harmonic analysis of locally compact groups. In the project you will establish this connection and apply it to specific Banach algebras such as the Banach algebra of compact operators on a Hilbert space.

6.3 Magdalena Musat
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Relevant interests:
Banach Spaces, Functional Analysis, Operator Algebras, Probability Theory

Suggested projects:

- **Geometry of Banach spaces** [Analysis 3]
  A number of very interesting problems concerning the geometry of Banach spaces can be addressed in a bachelor project. For example, does every infinite dimensional Banach space contain an infinite dimensional reflexive subspace or an isomorphic copy of $l_1$ or $c_0$? Or, does there exist a reflexive Banach space in which neither an $l_p$-space, nor a $c_0$-space can embed? Another project could explore the theory of type and cotype, which provides a scale for measuring how close a given Banach space is to being a Hilbert space.

- **Convexity in Banach spaces** [Analysis 3]
  The question of differentiability of the norm of a given Banach space is closely related to certain convexity properties of it, such as uniform convexity, smoothness and uniform smoothness. This project will explore these connections, and study further properties of uniformly convex (respectively, uniformly smooth) spaces. The Lebesgue spaces $L_p$ ($1 < p < \infty$) are both uniformly convex and uniformly smooth.

- **Haar measure** [MI]
  This project is devoted to the proof of existence and uniqueness of left (respectively, right) Haar measure on a locally compact topological group $G$. For example, Lebesgue measure is a (left and right) Haar measure on $\mathbb{R}$, and counting measure is a (left and right) Haar measure on the integers (or any group with the discrete topology).
• **Fernique’s theorem** | **SAND 1, Analysis 3** |
This project deals with probability theory concepts in the setting of Banach spaces, that is, random variables taking values in a (possibly infinite dimensional) Banach space. Fernique’s theorem generalizes the result that gaussian distributions on $\mathbb{R}$ have exponential tails to the (infinite dimensional) setting of gaussian measures on arbitrary Banach spaces.

6.4 Ryszard Nest
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**Relevant interests:**
Non-Commutative Geometry, Deformation Theory, Poisson Geometry

**Suggested projects:**

- **Clifford Algebras** | **LinAlg, Geom 1** |
Clifford algebra is a family $\mathcal{C}^{p,q}$ of finite dimensional algebras associated to non-degenerate bilinear forms which play very important role in both topology and geometry. The simplest examples are $\mathbb{R}$, $\mathbb{C}$ and the quaternion algebra $\mathbb{H}$. The main result is the periodicity modulo eight of $\mathcal{C}^{p,q}$, which has far reaching consequences (e.g., Bott periodicity, construction of Dirac operators) in various areas of mathematics.

- **Axiom of choice and the Banach-Tarski paradox** | **LinAlg, Analysis 1** |
The axiom of choice, stating that for every set of mutually disjoint nonempty sets there exists a set that has exactly one member common with each of these sets, is one of the more "obvious" assumptions of set theory, but has far reaching consequences. Most of modern mathematics is based on its more or less tacit assumption. The goal of this project is to study equivalent formulations of the axiom of choice and some of its more exotic consequences, like the Banach-Tarski paradox, which says that one can decompose a solid ball of radius one into five pieces, and then rearrange those into two solid balls, both with radius one.

- **Formal deformations of $\mathbb{R}^{2n}$** | **LinAlg, Geom 1** |
The uncertainty principle in quantum mechanics says that the coordinate and momentum variables satisfy the relation $[p,x] = \hbar$, where $\hbar$ is the Planck constant. This particular project is about constructing associative products in $C^\infty(\mathbb{R}^{2n})[[\hbar]]$ satisfying this relation and studying their properties.

6.5 Mikael Rørdam
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Relevant interests:

*Operator Algebras, Topics in Measure Theory, Discrete Mathematics*

Suggested projects:

- **Topics in $C^*$-algebras** [Analysis 3]
  $C^*$-algebras can be defined either abstractly, as a Banach algebra with an involution, or concretely, as subalgebras of the algebra of bounded operators on a Hilbert space. They can be viewed as non-commutative analogues of spaces, since every commutative $C^*$-algebra is equal to the set of continuous functions on a locally compact Hausdorff space. Several topics concerning $C^*$-algebras and concerning the study of specific examples of $C^*$-algebras, can serve as interesting topics for a bachelor project.

- **Topics in measure theory** [MI]
  We can here look at more advanced topics from measure theory, that are not covered in MI, such as existence (and uniqueness) of Lebesgue measure, or more generally of Haar measure on locally compact groups. Results on non-measurability are intriguing, perhaps most spectacularly seen in the Banach-Tarski paradox that gives a recipe for making two solid balls of radius one out of a single solid ball of radius one!

- **Topics in discrete mathematics** [Dis2 & Graf]
  One can for example study theorems about coloring of graphs. One can even combine graph theory and functional analysis and study $C^*$-algebras arising from graphs and the interplay between the two (in which case more prerequisites are needed).

Previous projects:

- **Irrational and rational rotation $C^*$-algebras** [Analyse 3]
- **Convexity in functional analysis** [Analyse 3]
- **The Banach-Tarski Paradox** [MI recommended]

6.6 **Thomas Vils Petersen**

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Relevant interests:

*Functional analysis, analysis, Banach algebras of functions.*
Suggested projects:

- **Convolution algebras** [An2, MI]
  These are Banach algebras of functions, and the product is the convolution product. Some possible topics:
  
  - Derivations on $L^1[0, 1]$.
  - Homomorphisms between weighted convolution algebras $L^1(\omega)$ on the half-line $\mathbb{R}^+$. 

7 Topology

7.1 Jesper Grodal

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Relevant interests:

Topology, Algebra, Geometry.

Suggested projects:

- **Group cohomology** [Alg2]
  To a group $G$ we can associate a collection of abelian groups $H^n(G)$, $n \in \mathbb{N}$, containing structural information about the group we started with. The aim of the project would be to define these groups, examine some of their properties, and/or examine applications to algebra, topology, or number theory. See e.g.: K.S. Brown: Cohomology of groups

- **Group actions** [Top, Alg2]
  How can groups act on different combinatorial or geometric objects? E.g. which groups can act freely on a tree? See e.g.: J.-P. Serre: Trees.

- **The Burnside ring** [Alg2]
  Given a group $G$ we can consider the set of isomorphism classes of finite $G$-sets. These can be "added" and "multiplied" via disjoint union and cartesian products. By formally introducing additive inverses we get a ring called the Burnside ring. What’s the structure of this ring and what does it have to do with the group we started with? See:

- **The classification of finite simple groups** [Alg2]
  One of the most celebrated theorems in 20th century mathematics gives a complete catalogue of finite simple groups. They either belong to one of three infinite families (cyclic, alternating, or classical) or are one of 26 sporadic cases. The aim of the project is to explore this theorem and perhaps one or more of the sporadic simple groups. See:

- **The Platonic solids and their symmetries** [Top, Alg2]
  A Platonic solid is a convex polyhedron whose faces are congruent regular polygons, with the same number of faces meeting each vertex. The ancient greeks already knew that there were only 5 platonic solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The aim of the project is to understand the mathematics behind this. See: [http://en.wikipedia.org/wiki/Platonic_solid](http://en.wikipedia.org/wiki/Platonic_solid)
• **Topological spaces from categories** [Top, Alg2]
  Various algebraic or combinatorial structures can be encoded via geometric objects. These "classifying spaces" can then be studied via geometric methods. The goal of the project would be to study one of the many instances of these this, and the project can be tilted in either topological, categorical, or combinatorial directions. See e.g.: A. Björner, Topological methods. Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.

• **Simplicial complexes in algebra and topology** [Alg1, Top]
  The goal of this project is to understand how simplicial complexes can be used to set up a mirror between notions in topology and algebra. For instance, the algebraic mirror image of a topological sphere is a Gorenstein ring.

**Previous projects:**
• **Steenrod operations—construction and applications** [AlgTopII]
• **Homotopy theory of topological spaces and simplicial sets** [AlgTopII]
• **Automorphisms of $G$ - with applications to group extensions** [AlgTopII, CatTop]

**7.2 Ib Madsen**
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**Relevant interests:**
*Homotopy theory, topology of manifolds.*

**Suggested projects:**
• **De Rham cohomology** []
• **Poincaré duality** []
• **Covering spaces and Galois Theory** []
• **The Hopf invariant** []

**7.3 Jesper Michael Møller**
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Relevant interests:

All kinds of mathematics.

Suggested projects:

- **Poincaré sphere** [Topology, group theory]
  What are the properties of the Poincaré sphere?

- **Topological combinatorics** [Dis1, Top]
  Combinatorial problems, such as determining chromatic numbers of graphs, can be solved using topological methods.

- **Partially ordered sets** [Dis1]
  Partially ordered sets are fundamental mathematical structures that lie behind phenomena such as the Principle of Inclusion-Exclusion and the Möbius inversion formula.

- **Chaos** [General topology]
  What is chaos and where does it occur?

- **Project of the day** [Mathematics]
  http://www.math.ku.dk/~moller/undervisning/fagprojekter.html

7.4 Nathalie Wahl

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Relevant interests:

Graphs, surfaces, manifolds, knots, algebraic structures.

Suggested projects:

- **Knots** [Alg1, Top]
  Mathematically, knots are embeddings of circles in 3-dimensional space. They are rather complicated objects that can be studied combinatorially or via 3-manifolds. The project consists of learning some basics in knot theory. See for example http://www.earlham.edu/~peters/knotlink.htm.

- **Braid groups, configuration spaces and links** [Alg1, Top]
  The braid group on $n$ strands can be defined in terms of braids (or strings), or as the fundamental group of the space of configurations of $n$ points in the plane. It is related to knots and links, and also to surfaces. The project consists of exploring braid groups or related groups like mapping class groups. See for example J. Birman, Braids, links, and mapping class groups.
• **Classification of surfaces** [Top, Geom1]
  Closed 2-dimensional surfaces can be completely classified by their genus (number of holes). There are several ways of proving this fact and the project is to study one of the proofs. See for example W. Massey, A Basic Course in Algebraic Topology, or A. Gramain, Topology of Surfaces.

• **3-manifolds** [Top, Geom1]
  3-dimensional manifolds are a lot harder to study than 2-dimensional ones. The geometrization conjecture (proved recently by Perelman) gives a description of the basic building blocks of 3-manifolds. Other approaches to 3-manifolds include knots, or “heegaard splittings”, named after the Danish mathematician Poul Heegaard. The project consists of exploring the world of 3-manifolds. See for example [http://en.wikipedia.org/wiki/3-manifolds](http://en.wikipedia.org/wiki/3-manifolds).

• **Non-Euclidean geometries** [Geom1]
  Euclidean geometry is the geometry we are used to, where parallel lines exist and never meet, where the sum of the angles in a triangle is always 180°. But there are geometries where these facts are no longer true. Important examples are the hyperbolic and the spherical geometries. The project consists of exploring non-euclidian geometries. See for example [http://en.wikipedia.org/wiki/Non-euclidean_geometries](http://en.wikipedia.org/wiki/Non-euclidean_geometries).

• **Frobenius algebras, Hopf algebras** [LinAlg, Alg1]
  A Frobenius algebra is an algebra with extra structure that can be described algebraically or using surfaces. A Hopf algebra is a similar structure. Both types of algebraic structures occur many places in mathematics. The project consists of looking at examples and properties of these algebraic structures. See for example J. Kock, Frobenius algebras and 2D topological quantum field theories.

• **Khovanov Homology** [AlgTop – or familiarity with category theory]
  The complexity of knots is immense. Explore [http://katlas.org/](http://katlas.org/). Over the last 100 years various tools have been developed to distinguish and classify knots. A lot of work is still needed to have a good understanding of the world of knots. This project would aim at understanding one of the stronger tools available to this date; Khovanov Homology.

• **Operads and Algebras** [Alg2]
  Operads is an effective tool to cope with exotic algebraic structures. How do you for instance work with algebraic structures that are not (strictly) associative? Depending on interest, the project can have a more algebraic or more topological flavor.

• **Morse Theory** [Geom2 – for instance simultaneously]
  The second derivative test, known from MatIntro, tells you about local behaviours of a 2-variable function. Expanding this test to manifolds in general yields Morse Theory, which plays a key role in modern geometry.
  This project would start out by introducing Morse Theory. Various structure and classification results about manifolds could be shown as applications of the theory.
8 History and philosophy of mathematics

8.1 Jesper Lützen
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Relevant interests:

History of Mathematics

Suggested projects:

- The history of non-Euclidean geometry [Hist1, preferably VtMat]
  How did non-Euclidean geometry arise and how was its consistency "proved".
  How did the new geometry affect the epistemology of mathematics?

- The development of the function concept [Hist1]
  How did the concept of function become the central one in mathematical
  analysis and how did the meaning of the term change over time.

- Archimedes and his mathematics [Hist1]
  Give a critical account of the exciting life of this first rate mathematician
  and analyze his "indivisible" method and his use of the exhaustion method.

- What is a mathematical proof, and what is its purpose [Hist1, VtMat]
  Give philosophical and historical accounts of the role(s) played by proofs
  in the development of mathematics

Previous projects:

- A brief history of complex numbers [Hist1, preferably KomAn]

- Mathematical induction. A history [Hist1]

- Aspects of Euler’s number theory [Hist1, ElmTal]

- Mathematics in Plato’s dialogues [Hist1, VtMat]

- Axiomatization of geometry from Euclid to Hilbert [Hist1, preferably VtMat]

- Lakatos’ philosophy applied to the four color theorem [Dis, Hist1]

- History of mathematics in mathematics teaching: How and why
  [Hist1, DidG preferably DidMat]

8.2 Jan Philip Solovej
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Relevant interests:

*Mathematical Physics, Quantum Mechanics, Spectral Theory, Partial Differential Equations*

Suggested projects:

- **Partial Differential equations: The wave equation and the heat equation [An1,An2]**
  The goal of the project is to derive the solution formulas for the initial value problems for the wave equation (homogeneous and inhomogeneous) and the heat equation. Moreover, the project will focus on the strong and weak Huygens principles for the wave equation and on the maximum principle for the heat equation.
  

- **First order partial differential equations [An1,An2]**
  The project considers first order partial differential equations, in particular, quasi-linear equations. The goal is to describe the method of characteristics and to study equations that exhibit shock formation.
  
  Literature: F. John, Partial Differential Equations

- **Why you cannot hear the shape of a drum [An1,An2]**
  For any compact open set in \( \mathbb{R}^n \) we can define the discrete set of eigenvalues of the Dirichlet Laplacian. In the 2-dimensional case these are the frequencies you would hear if the domain is played as a drum. Translating, rotating or reflecting a domain does not change the eigenvalues. But is this the only way domains can have the same frequencies? i.e., can you hear the shape of a drum? The answer is no and the project constructs pairs of domains in \( \mathbb{R}^2 \) that have the same Dirichlet Laplace eigenvalues, but are not isometric. As part of the project the existence of the Dirichlet Laplace eigenvalues will be constructed. A certain regularity property of eigenfunctions will have to be used, but not proved.
  
  Literature: Notes and exercises and the article Buser, Conway, Doyle, and Semmler, Some planar isospectral domains, IMRN, 1994 (9)

- **Poisson summation formula and Gauss circle problem [An1]**
  The aim is to prove the Poisson summation formula using the theory of Fourier series. The Poisson summation formula can be used to estimate how close Riemann sums are to Riemann integrals. This will be generalized to functions of several variables and applied to count the integer points in a large ball (Gauss circle problem). The problem is of interest in number theory and in quantum mechanics, where it may be interpreted as the number of states in a Fermi gas below the Fermi level.
  
  Literature: Notes and exercises and the article Buser, Conway, Doyle, and Semmler, Some planar isospectral domains, IMRN, 1994 (9)

- **Estimating eigenvalues of Schrödinger operators [An1,An2]**
  The goal of this project is to estimate the number and the sum of negative eigenvalues for Schrödinger operators. In quantum mechanics the negative eigenvalues of Schrödinger operators represent bound states of quantum systems and
their sum represents the total energy of the bound states. This project requires first to introduce the Fourier transform
9 Set Theory

9.1 Asger Törnquist

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Relevant interests:

Mathematical logic, descriptive set theory, ergodic theory, operator algebras.

Suggested projects:

- **Classical and effective descriptive set theory**
  Classical descriptive set theory is concerned with the basic properties of sets and functions that arise in analysis, e.g. Borel sets. Effective descriptive set theory is a modern refinement of this theory which combines ideas from computability theory with the classical ideas. It is a powerful tool which has been used to prove surprising structure theorems about the structure of Borel equivalence relations and Borel graphs. A possible goal for a project could be to prove Silver’s dichotomy theorem for Borel equivalence relations, which says that any Borel equivalence on $\mathbb{R}$ either must have countably many classes, or there must be a homeomorphic copy of the Cantor set which meets every class in at most one point.

  [The project requires some basic knowledge of metric spaces and general topology. Some knowledge of measure theory is desirable.]

- **Axiomatic set theory and Gödel’s constructible universe**
  This project is about the Zermelo-Fraenkel system of axioms for set theory (ZFC), which many today accept as the basic framework for all of mathematics. A goal of the project could be to introduce Gödel’s constructible universe $L$, and use this to prove the following: If ZFC is consistent (that is, one cannot deduce a contradiction from it), then so is ZFC in conjunction with the Continuum Hypothesis ($2^{\aleph_0} = \aleph_1$). This means that the continuum hypothesis cannot be disproved from ZFC (if ZFC is consistent), and cannot introduce a contradiction, if there isn’t one there already.

- **Ergodic theory and orbit equivalence**
  Ergodic theory is concerned with the study of measure preserving actions of groups on measure spaces. Each such action induces an orbit equivalence relation (Danish: Baneækvivalensrelation). It is of great interest to study how much the orbit equivalence relation in itself “remembers” about the group and the action. A goal of the project could be to prove Dye’s theorem: All “ergodic” (that is, irreducible, in some sense) actions of $\mathbb{Z}$ on the unit interval $[0, 1]$ which preserve Lebesgue measure produce essentially the same equivalence relation, up to a natural notion of isomorphism known as orbit equivalence. A more ambitious goal would be to also prove Hjorth’s theorem, which stands in sharp contrast to Dye’s: Every countably infinite discrete group with property (T) (e.g. $\text{SL}_3(\mathbb{Z})$) has uncountably many orbit inequivalent measure preserving ergodic actions on $[0, 1]$. 
The project requires some knowledge of measure theory, of metric spaces, and, for Hjorth’s theorem, some elementary knowledge of operators on Hilbert spaces.
10  Applied mathematics

10.1  Elisenda Feliu

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Relevant interests:

*Applied algebraic methods, Models in biology, Differential equations.*

Suggested projects:

- **Algebraic methods in biology. [Alg2]**
  The project consists of learning how to model biochemical systems using polynomial equations and the basic properties of the models. The project involves also the hands-on analysis of some specific model (that will be chosen from the literature together with the student).

- **Numerical algebraic geometry. [Alg2]**
  The student will learn homotopy-based methods for numerically solving polynomial equations. The techniques will be applied to a real-world problem, that either the student or the teacher will propose.

- **Qualitative behaviour of models of differential equations. [An1]**
  The project consists of the study of techniques to analyse the qualitative behaviour of differential equations models. The techniques will be applied to analyse a specific model, preferably in biology, that either the student or the teacher will propose.

10.2  Carsten Wiuf

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Relevant interests:

*Markov chains applied to biochemical processes, Simulation, ODEs.*

Suggested projects:

- **Mathematical analysis of biochemical systems at and near equilibrium. [Diff, Stok]**
  The project consists of a theoretical part and an applied part. The theoretical part consists in learning how to describe a system as a Markov chain (stochastic) and a system of ODEs (deterministic). The applied part of the project analyzes a biochemical system as a Markov chain and an ODE system and contrasts the results with each other. The biochemical system will be chosen from the literature together with the student. The applied part involves implementation of small scripts in R or Maple.
11 Mathematics of Quantum Theory

11.1 Matthias Christandl
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Relevant interests:
*Quantum Information Theory and Quantum Computation*

Suggested projects:

- **Bell inequalities [QIT1]**
  Quantum mechanics violates Bell’s inequalities, which are inequalities that classical local structures satisfy. When the violation is very high, a stronger notion, known as *rigidity* can appear, where it is possible to infer the structure of measurements and states that have been used to infer the violation. The project will investigate rigidity, taking its starting point in a famous theorem by Tsirelson.

- **Quantum Entropy [QIT2]**
  Quantum entropy or von Neumann entropy is a concave function of the quantum state. Interestingly, there are more concavity-like properties that quantum entropy satisfies. It is the aim of the project to study these properties.

- **Quantum Tomography [QIT3]**
  Quantum tomography is the art of inferring a quantum state from measured data. To a large extent, quantum tomography is a problem from classical statistics and it is the goal to collect classical methods to treat this problem.
12 Other areas

12.1 Discrete mathematics
Projects in this area can be found with
- Bergfinnur Durhuus (4.1)
- Søren Eilers (6.1)
- Jørn B. Olsson (??)
- Mikael Rørdam (6.5)

12.2 Teaching and didactics in mathematics
Projects in this area can be found with
- Niels Grønbæk (6.2)
- Jesper Lützen (8.1)

12.3 Aspects of computer science
Projects in this area can be found with
- Søren Eilers (6.1)
- Jens Hugger (4.2)
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