Entropy inequalities

Points in Δ lie above the lines connecting neighbouring points Q_k. Thus:

$$H(P) \geq \alpha_k - \beta_k \mathrm{IC}(P)$$

with

$$\begin{aligned} \alpha_k &= \ln k + (k+1)\ln\left(1+\frac{1}{k}\right), \\ \beta_k &= k(k+1)\ln\left(1+\frac{1}{k}\right). \end{aligned}$$

Proof. Fix P and $k \geq 1$. Define $f : [0,1] \to \mathbb{R}$ by

$$f(x) = -\ln x + \beta_k x^2 - \alpha_k x.$$

Then f is a concave/convex function, so what we have to check is that $F(P) \geq 0$. It suffices to prove this for a mixture of neighbouring uniform distributions. ... easy! $\square$