Lemma of replacement

Assume that $f : [0,1] \to \mathbb{R}$ *is concave/convex, i.e. for some* $0 < x < 1$, f *is concave in* $[0,x]$, *convex in* $[x,1]$. *For* $P \in M_+^1(n)$ *put*

$$F(P) = \sum_{i=1}^{n} f(p_i) .$$

Then, to any $P \in M_+^1(n)$ *there exists* $P_0 \in M_+^1(n)$ *and* $P_1 \in M_+^1(n)$ *such that*

$$F(P_0) \le F(P) \le F(P_1)$$

and such that P_1 *is a mixture of* U_1 *and* U_n *and* P_0 *a mixture of* U_k *and* U_{k+1} *for some* $1 \le k \le n-1$.

(U's denote uniform distributions).