

# Paradigms of cognition – an informtion theoretical analysis

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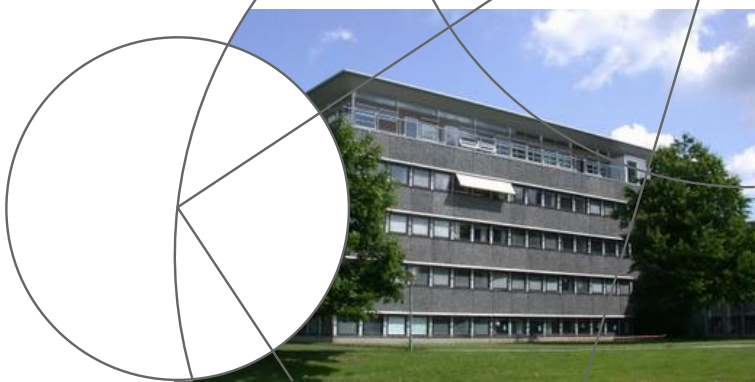

# philosophy?

Well, yes, to some extent. We will deal with concepts such as truth, belief and knowledge.

And we do find it important to see things from a higher perspective. This will steer the course and leed to valuable interpretations.

But, in contrast to a course in philosophy we insist that the analysis gives rise to quantitative considerations – and precise mathematical results.

# perhaps lectures will start like this

The whole is the world , $\Omega$

Situations from the world involve Nature and you, Observer (statistician, physicist, economist or ...).

Nature has no mind but holds the truth $(x)$,

Observer has a creative mind,

- seeks the truth $(x)$
- is confined to belief $(y)$
- aims at knowledge $(z)$.

Knowledge is

- the synthesis of extensive experience
- an expression of how Observer perceives situations from $\Omega$
- how truth manifests itself to Observer, to you.

# where does the quantitative element come in?

Short answer: Via description and the view that this requires effort. We will use $\Phi(x, y)$ for the effort in a situation where the truth is $x$ and you believe $y$.

A key principle which will help Observer to choose sensible methods of description is the perfect match principle:

$\Phi(x, y) \geq \Phi(x, x)$ with equality only if $y = x$.

If $x's$ and $y$'s are probability distributions, you may have met this principle before. But we will work quite abstractly, e.g. $x$'s and $y$'s could be points in Hilbert space.

# which mathematical tools are essential?

Only rather elementary ones. Some key results depend heavily on game theory , but you need not know game theory beforehand. It is an integral part of the course to develop elements of game theory from scratch. This will be great fun! And for the area we will address, that of two-person zero-sum games we will extend standard setting and introduce new methods, enabling various shortcuts.

For some of this, convexity is important, for some, basic elements of general topology . But for key applications it is necessary that you know basics of probability theory.

# ... and the exercise sessions?

I may start asking you to study, what turns out to be the classical Pythagorean theorem. This will be given an unorthodox formulation which will later enable the proof of a key result related to Baysean inference. Further notions from statistics which we will come across relates to exponential families ...