Extreme Value Theory for Space-Time Processes with Heavy-Tailed Distributions

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Many real-life time series often exhibit clusters of outlying observations that cannot be adequately modeled by Gaussian distributions. Heavy-tailed distributions such as the Pareto distribution have proved useful in modeling a wide range of bursty phenomena that occur in areas as diverse as finance, insurance, telecommunications, meteorology, and hydrology. Regular variation provides a convenient and unified background for studying multivariate extremes when heavy tails are present. In this paper, we study the extreme value behavior of the space-time process given by

$$X_t(s) = \sum_{i=0}^{\infty} \psi_i(s) Z_{t-i}(s),$$

$s \in [0,1]^d$, where $\{Z_t, t = 0, \pm 1, \pm 2, \ldots,\}$ is an iid sequence of random fields on $[0,1]^d$ with values in the Skorokhod space $D([0,1]^d)$ of càdlàg functions equipped with the J_1-topology. The coefficients ψ_i's are deterministic continuous real-valued fields on $[0,1]^d$. The indices s and t refer to a measurement taken at location s at time t. For example, $X_t(s), t = 1, 2, \ldots,$ could represent the time series of annual maxima of ozone levels at a location s. The problem of interest is determining the probability that the maximum ozone level over the entire region $[0,1]^2$ does not exceed a given standard level $f \in D([0,1]^2)$ in n years. By establishing a limit theory for point processes based on $\{X_t(s), t = 1 \ldots, n\}$, we are able to provide approximations for probabilities of extremal events. This theory builds on earlier results of de Haan and Lin (2001) and Hult and Lindskog (2003) for regular variation on $D([0,1]^d)$ and Davis and Resnick (1985) for extremes of linear processes with heavy-tailed noise.