Mathematicians are usually tempted to meet a challenge. The point of competition is the challenge of doing better then the competitors. It works like any tournament of e.g. chess. In recreational mathematics the challenge is interior, to figure out what was in the poser's mind. A little like cross–word puzzles, what is hidden here.

The psychology works similarly, just because a problem is posed in a journal, it is by definition a challenge. Many people will feel the temptation to try their powers in problem solving, and the mathematically gifted may solve the problem. In contrast to the competitions, the time limit is practically non–existing. It is rather a question of losing patience.

I have offered problems monthly for about 12 years, and the audience I have in mind is mainly highschool students, some young college students too, and in real life, retired closet mathematicians to some extend. This means that my problems are more elementary than those used in competitions, I do avoid differential– and integral calculus. This means, that if a problem is easily solved by these means, it must have a more elementary solution. The fun is, that the students write to me, that my solution is not elegant, as they may solve the problem by differentiation. Well, I find it a success that they have been able to use what they have learned, and it is too much to ask that they should appreciate my solution, given to explain the result for younger readers not knowing calculus yet.

I did once offer the Martin Gardner problem of the brace. It requires a little calculus, but it worked well in the way, that highschool teachers told me, that the appearance of the problem in my magazine was strongly motivating the pupils to learn integration. Unexpected coordination!

But a problem I find ideal is the following. Make a hexagon–pattern in a hexagon and count all hexagons in the figure of any size. The result is always a third power. Why?

Well, the pattern is actually a projection of a sliced cube, so there is a natural correspondence between the visible hexagons and the small cubes, of which there must be a third power. So, it is just a question of point of view, rather than technique of proving formulas.

Another favorite is the Fermat problem.

Some problems we appreciate in mathematics are not too good for recreational purposes; problems without solution. Take e.g. Dudeney's problem of 3 facilities and 3 houses. I prefer a positive answer, so I try the problem on a torus. But then I may expand the question to 4 of each and still get a solution!

In the famous problem of crossing a desert with a jeep with limited supply of fuel by making deposits, one requires a section of the harmonic series, so called harmonic numbers. I asked the question, "is there a limit to the size of the desert, we are able to cross?" I received a letter from Norway that a reader was so amazed by the answer, that he has teased every friend of his with this question and answer.

I do feel the pleasure of solving problems myself. E.g. a strange summation due to Ira Gessel was posed in Monthly in 1995,

$$S(n) = \sum_{3k \le n} 2^k \frac{n}{n-k} \binom{n-k}{2k}$$

Solution:. *For all $n \in \mathbb{N}$, we have*

$$S(n) = \sum_{3k \le n} 2^k \frac{n}{n-k} \binom{n-k}{2k} = 2^{n-1} + \cos\left(n \cdot \tfrac{\pi}{2}\right)$$

The fun was not only to see the proof in print with the added title, "A Surprisingly Simple Summation Solution," but also to see others' e.g. Donald Knuth doing something else and then "continuing as Andersen and Larsen."

But certain aspects of mathematics are difficult to reach. The power of generalization. It may be sometimes easier to solve a more general problem, but not often. As a very peculiar example of my own let me show you a couple of strange formulas from Kaucký's collection of identities,

$$\sum_{k=1}^{2n-1} (-1)^{k-1} \binom{2n}{k}^{-1} \sum_{j=1}^{k} \frac{1}{j} = \frac{n}{2(n+1)^2} + \frac{1}{2n+2} \sum_{k=1}^{2n} \frac{1}{k}$$

$$\sum_{k=1}^{2n-1} (-1)^{k-1} \binom{2n-1}{k}^{-1} \sum_{j=1}^{k} \frac{1}{j} = \frac{2n}{2n+1} \sum_{k=1}^{2n} \frac{1}{k}$$

Now, these sums look as definite sums, but they are actually special cases of one indefinite sum, with x any complex number,

$$\sum (-1)^k \binom{x}{k}^{-1} \sum_{j=1}^{k} \frac{1}{j} \delta k = \frac{(-1)^k k}{x+2} \binom{x}{k-1}^{-1} \left(\frac{1}{x+2} - \sum_{j=1}^{k} \frac{1}{j} \right)$$

And because of the fact that the sum is indefinite, it is trivial! The proof consists in taking the difference of the closed expression. (Just as we prove an indefinite integral by differentiation.) Usually this aspect is most difficult to reach, so recreational mathematics do not lead every addict to appreciate the depth of the subject.