PENTOMINO[1] BATTLESHIPS

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August 30, 2007

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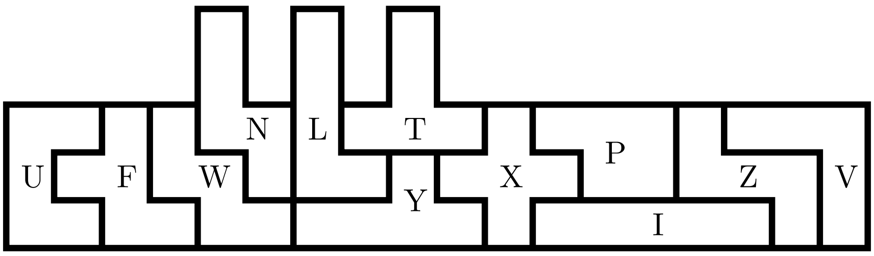
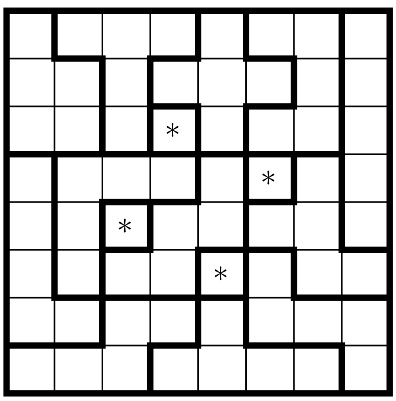
Nostalgia. In the 60s I first learned about pentominoes from Martin Gardner [1] Chapter 13, Polyominoes[1]. Especially the figures on pages 119-120 were appealing: Playing with them trying to occupy most of a chessboard was great fun. In order to explain the fun, we need to give them names. Traditionally they are:



The figures on fig. 72 were inspiring: Look at the first:



[1]Registered Trademark of Solomon W. Golomb [3]

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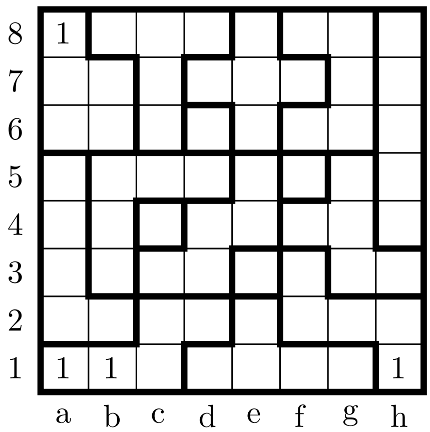
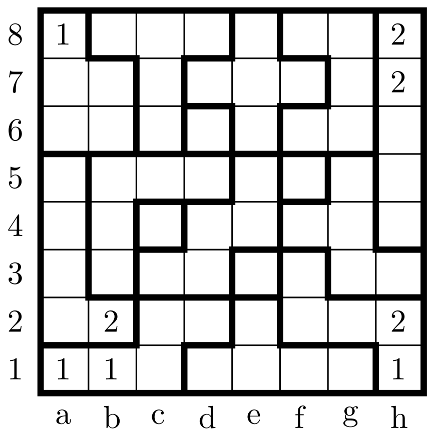
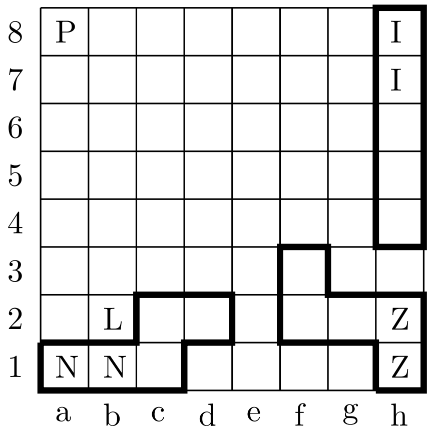
Battleships. Filling the most of a chessboard with 64 squares with the 12 pentominoes covering 60 of them leaves 4 empty squares. Now, we made this into a variation of the game of battleships. We shot at the chess board with a volley of 4 shots aiming at hitting the empty squares. Of course, we did usually hit some of the pentominoes. But after maybe 4 volleys we were able to locate them and shoot a volley hitting all 4. The answer to the volley would be how many shots did hit which pentomino. E.g. let's shoot in the picture above:



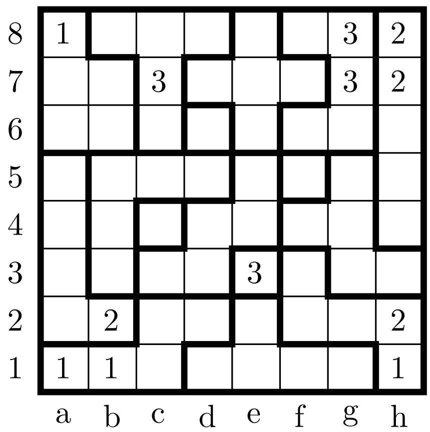
We would say, I hit a1, b1, a8 and h1. The answer should be, you have hit twice the N, once in Z and once in P. This information allows the location of the N, but with two possible orientations and that the other two are in the corners. So, to obtain more information, it would be tempting to shoot the second volley something like b2, h2, h7 and h8.



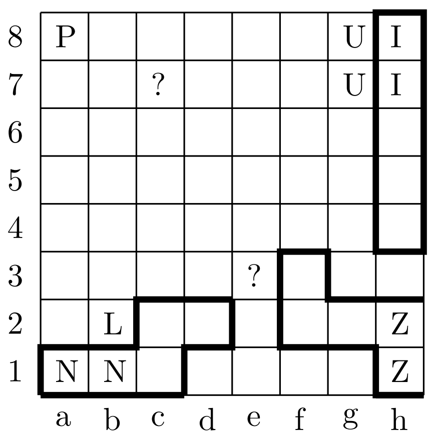
Now we learn, that we have hit twice the I, the L and the Z. This information allows us to place the N, the I and the Z, but the P still has 6 possible orientations. And the L has 5 ways to orient.



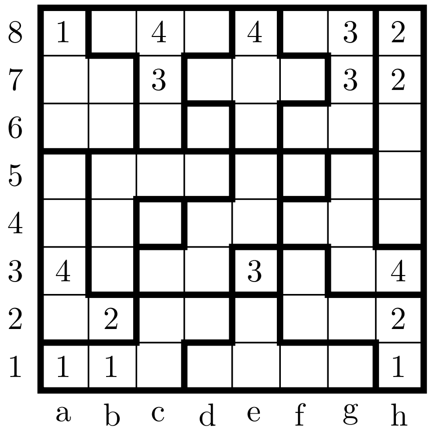
Next volley may be used to locate some missing orientation. E.g. e3 for a possibility for the L and to possibly show that the Y is between the N and the Z. c7 for a possible place of the P and g7 and g8 to find another piece.



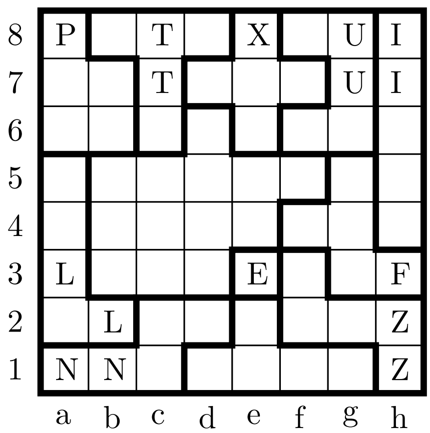
This time we learned, that the U was hit twice, the T and an empty was hit once. So, we may reach a few conclusions, P has 5 ways left, the U has 4 ways, either the Y is as shaped between the N and the Z, or the T is located here with g1 as empty. And the L has only 4 ways. Not much! And the empty square may be one of two.



Now it is confusing, and we may try a3, c8, e8 and h3.



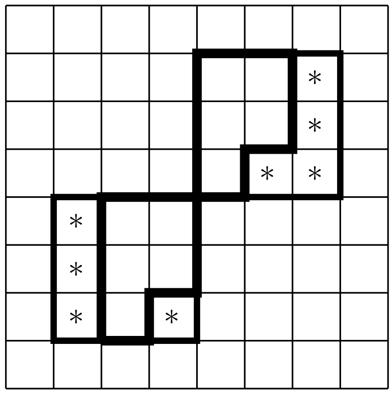
We get the information, that we have hit L, T, X and F. Then T and E are located as c8 and e3 respectively. And L is placed, the F must be in h3, and U has two ways, but with the X in e8, one is the only.



Only V and W are left, but though V has only one place, (the one we know), W may be placed in two ways, leaving either c3 or e5 empty. And reflecting PV mey move the empty space from d5 to d8. So we must shoot a fifth time to be sure!

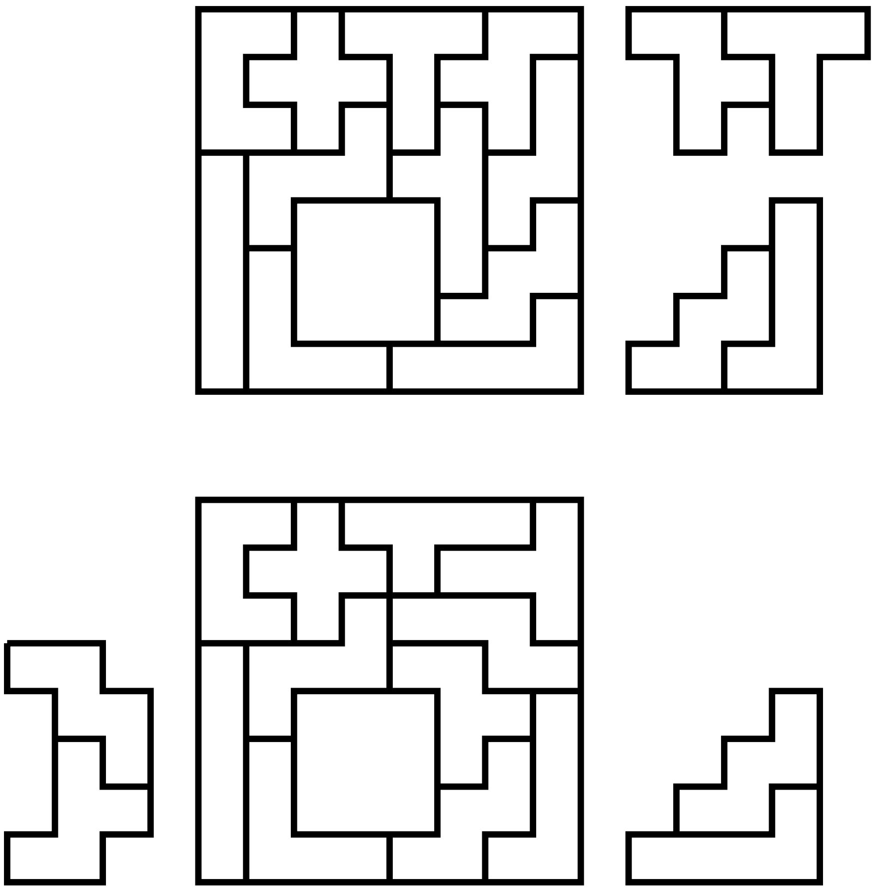
We have learned something inspiring. Although we have placed 11 figures, we still have to decide the exact location of the last, the W in the example. To make the game harder, we should look for such ambiguities.

Antisymmetry of the P. The P gives rise to the greatest ambiguity, placed in a 3×3–square it has 16 different ways to distribute the empty four!

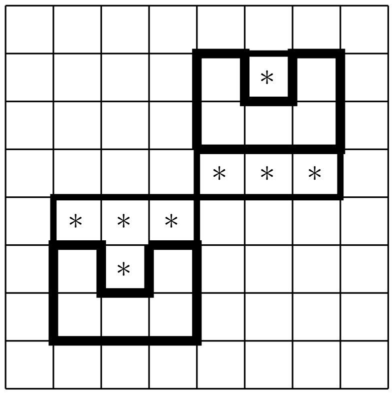


Of course, it adds to the trouble to place this square as far from the corner as possible. So, we have the following problem: Fill in the board with the 11 other pentominoes leaving a square including most of the center to the P.

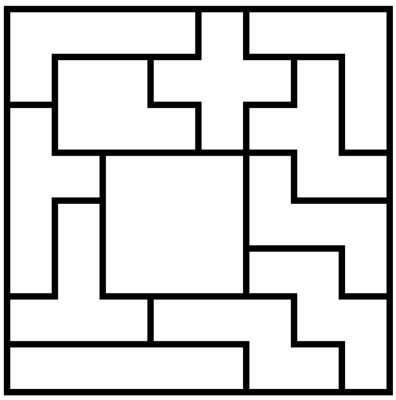
My 7 solutions.



Alternative: Symmetry and Translation of the U. The U is symmetric, so turning it around only gives 4 different orientations. But, inside the 3×3–square is room for translation giving rise to 4 new different orientations, leaving us with 8 different distributions of the 4 empty squares.



And for this piece I have been able to place the square including the center of the board:



If I am right in finding exactly one solution, the search for ambiguity have reach a case of uniqueness, that this pattern can be solved by these 11 pieces in one and only one way.

REFERENCES

1. Martin Gardner, *Mathematical Puzzles and Diversions from Scientific American,* G. Bell and Sons Ltd., London, 1961, 163 s.
2. Martin Gardner, *Morsom matematik,* Borgens Forlag, København, 1963, 173 s.
3. Solomon W. Golomb, *Polyominoes,* Princeton University Press, Princeton, 1994 (1. ed. 1965), 184 s.
4. George E. Martin, *Polyominoes A Guide to Puzzles and Problems in Tiling,* Mathematical Association of America, Washington D. C., 1991, 184 s.
5. Mogens Esrom Larsen, There Are No Holes Inside a Diamond, *Agate* 3 (1989) 30–36.