

**COMMENTS TO  
G. GRUBB: “DISTRIBUTIONS AND OPERATORS”**

Springer Verlag, New York 2009

**Corrections, updated May 6, 2017.**

*Notation:*  $x$  means page  $x$ , with  $x^y$  indicating line  $y$  from above,  $x_y$  indicating line  $y$  from below.

- 4<sup>15</sup> replace “differention” by “differentiation”
- 10<sup>6</sup> replace “ $f_2(x) \geq 0$  for all  $x$ ” by  $f_2(t) \geq 0$  for all  $t$ ”
- 13<sup>19+23</sup> replace “ $p_{j,k}$ ” by “ $p_{k,j}$ ”
- 14<sub>7</sub> replace “mapping  $T$ ” by “linear mapping  $T$ ” (as in Theorem B.18)
- 18<sup>12+18</sup> replace “ $L^1$ ” by “ $L_1$ ”
- 24<sub>8+7</sub> replace “ $i = 0$ ” by “ $j = 0$ ”
- 24<sub>7</sub> add the sentence “The conclusion of Theorem 2.17 also holds when the  $V_j$  are arbitrary open sets, since they can be replaced by bounded sets  $V_j \cap B(0, R)$  with  $R$  taken so large that  $K \subset B(0, R)$ .”
- 42<sup>5</sup> replace “ $(\varphi)$ ” by “ $(\check{\varphi})$ ”
- 42<sub>10</sub> replace “(2.35)” by “(2.32)”
- 44<sub>15</sub> replace “ $J \circ T^{-1}$ ” by “ $JT^{-1}$ ”
- 60<sup>13</sup> add the line “here  $\partial f = g$ .”
- 62<sub>14</sub> replace “(C.11)” by “(C.10)”
- 63<sup>4</sup> replace “ $v$ ” by “ $u$ ” in two places
- 64<sub>7</sub> replace “ $\chi_N u$ ” by “ $\chi_N u = \chi(x/N)u$ ”
- 65<sup>2+5+6</sup> replace “ $L^2$ ” by “ $L_2$ ” in the subscripts
- 65<sub>2</sub> replace “ $B(0, \frac{1}{j})$ ” by “ $B(x, \frac{1}{j})$ ”
- 66 let the footnote refer to (3.60) instead of (3.43)
- 72<sub>11</sub> change the definition of  $\tilde{v}_\delta$  to

$$\tilde{v}_\delta(x) = \tilde{u}\left(\frac{\alpha+\beta}{2} + \frac{1}{1-\delta}\left(x - \frac{\alpha+\beta}{2}\right)\right)$$

- 73<sup>9</sup> replace “periodic” by “periodic”
- 76<sup>16</sup> replace “ $m - 1$ ” by “ $m - 1$ ”
- 79<sub>9</sub> replace “ $dy_n dx'$ ” by “ $dx' dy_n$ ”
- 83<sub>9</sub> replace “ $\Omega_b = \{x \in \mathbb{R}^n \mid 0 \leq x_j \leq b\}$ ” by “ $\overline{\Omega}_b$ , where  $\Omega_b = \{x \in \mathbb{R}^n \mid 0 < x_j < b\}$ ”
- 84<sup>6</sup> replace “ $\Omega_R$ ” by “ $\Omega_b$ ”
- 84<sup>9</sup> replace “the lemma” by “Theorem 4.29”
- 89<sup>7</sup> replace “ $(H, V, l(u, v))$ ” by “ $(H, V, l_0(u, v))$ ”

- 121<sub>8</sub> replace " $(-1)^{m-1}(m-1)!$ " by " $\frac{(-1)^{m-1}}{(m-1)!}$ "  
 126<sub>14</sub> replace "Exercise 12.36" by "Exercise 12.35"  
 126<sub>10</sub> replace "this theorem" by "Theorem 6.3"  
 127<sub>1</sub> add "(The constant  $4/3$  can e.g. be found as the maximum of  $(1+2s+2t)/(1+s+t+st)$  for  $s=|x|^2$ ,  $t=|y|^2 \in \overline{\mathbb{R}}_+$ .)"  
 135<sub>4</sub> replace " $d\eta d\zeta$ " by " $d\zeta d\eta$ "  
 136<sub>17</sub> replace "when  $u$ " by "when  $\varphi$ "  
 141<sub>7-10</sub> replace "each  $\varphi \in C_0^\infty(\Omega)$ , that ... , and  $D_j\varphi \in C_0^\infty(\Omega)$ ." by "each  $\eta_l$  as in Corollary 2.14:

$$\eta_l D_j u = \eta_l D_j (\eta_{l+1} u) \in H^{s-1}(\mathbb{R}^n),$$

since  $\eta_{l+1} = 1$  on  $\text{supp } \eta_l$ , and here  $\eta_{l+1} u \in H^s(\mathbb{R}^n)$  implies  $\eta_l D_j (\eta_{l+1} u) \in H^{s-1}(\mathbb{R}^n)$  by Lemma 6.7."

- 158<sup>1</sup> replace "Show that" by "Let  $\text{Re } b > -2$ . Show that"  
 187<sup>17</sup> add the sentences "For  $u \in \mathcal{D}'(\Omega)$  one can obtain this by using a parametrix  $Q$  together with nested cutoff functions as in the proof of Theorem 6.29. The induction argument is to write

$$P\psi_{j+1}u = [P, \psi_{j+1}]\psi_j u + \psi_{j+1}Pu + \psi_{j+1}P(\psi_j - 1)u,$$

where  $[P, \psi_{j+1}]$  is of order  $d-1$ ,  $\psi_{j+1}Pu \in H^s$  and  $\psi_{j+1}P(\psi_j - 1)u \in C^\infty$ , so that application of  $Q$  lifts the regularity one step."

- 224<sup>11</sup> replace "belong to  $\mathcal{S}(\overline{\mathbb{R}}_+^n)$ " by "belongs to  $\mathcal{O}_M(\mathbb{R}^{n-1}, \mathcal{S}(\overline{\mathbb{R}}_+))$ "  
 224<sup>12</sup> replace "then also" by "then"  
 224<sub>8</sub> replace " $dx_n d\xi'$ " by " $dx$ "  
 226<sub>9-7</sub> " $\rho_{(m)}$ " should be " $\varrho_{(m)}$ " (such wrong fonts occur here and there in the book)  
 305<sub>13</sub> replace " $\mathcal{S}'(\mathbb{R})$ " by " $\mathcal{D}'(\mathbb{R})$ "  
 320<sup>15</sup> replace " $LiC^-$ " by " $C^-$ "  
 349<sub>3</sub> replace " $|p(\xi)| \leq C$  for  $\xi \in X$ " by " $|p(x)| \leq C$  for  $x \in \Omega$ "  
 350<sub>11</sub> replace " $\beta a x_2$ " by " $\beta a(x_2)$ "  
 352<sub>13</sub> replace "12.9 3<sup>o</sup>" by "12.9"  
 353<sub>5</sub> add the sentence "Moreover,  $H$  is dense in  $V^*$ ; this is seen e.g. by observing that the mapping  $f \mapsto \ell_f$  from  $H$  to  $V^*$  is the adjoint of the injection of  $V$  into  $H$ ; here one can apply Theorem 12.7."  
 359<sub>8</sub> replace "at" by "as"  
 362<sup>11</sup> replace " $e^{i\theta}$ " by " $e^{i\theta}$ "  
 368<sup>19+20</sup> remove "see in particular Exercise 4.14"  
 370<sup>23</sup> remove " $rr$ "  
 434<sup>5</sup> replace "Exercise B.1" by "Exercise B.3"  
 436<sub>1</sub> the signs " $|$ " are superfluous  
 437<sup>6-10</sup> The proof "In fact, if ... 1<sup>o</sup>." can be replaced by a shorter proof: "In fact, assume that for some  $x_0$ ,  $|\Lambda(x_0)| > (\varepsilon/\delta)p(x_0)$ ."

Then  $\Lambda(x_0) \neq 0$ , and  $p(|\Lambda(x_0)|^{-1}\varepsilon x_0) < \delta$ . By (\*), this implies  
 $\varepsilon > |\Lambda(|\Lambda(x_0)|^{-1}\varepsilon x_0)| = \varepsilon$ , a contradiction. This shows 1°."

437<sup>13</sup> the signs "|" are superfluous

448<sup>18</sup> replace "order  $m$ " by "order  $k$ "